A New Model of Subsurface Flow in an Unconfined Surface Soil Layer on an Irregular Hillslope

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1 Introduction

To better assess hillslope stability for landslide prediction, we would like to develop a three-dimensional model for shallow groundwater flow in a surface soil layer on an irregular hillslope. In terms of the assumption of shallow groundwater flow, we derived a new and Boussinesq-type perturbation solution of hydraulic head as well as a depth-averaged equation of groundwater table evolution. For numerical solutions, we used the leading-order evolution equation having a strong advection term, a nonlinear diffusion term and a source term. To tackle efficient and accurate calculation efficiency, we proposed a new and high resolution Godunov-type finite volume scheme with specific treatments to the nonlinear diffusion term for assuring the property of numerically well-balancing. Some cases are conducted for verification of the new model we proposed. This work is supposed to provide a new three-dimensional theory of groundwater motion and a corresponding numerical model.

2 Fundamental theory

Governing equations

We consider a thin sloping aquifer with the characteristic length $L = O(10)$ m and thickness $H = O(1)$ m. Applying the depth-averaging method with kinematic and dynamic boundary conditions yields the governing equation of phreatic surface evolution as

$$S\frac{\partial \eta}{\partial t} = \nabla \cdot (\eta - b)\nabla \eta + \gamma, \tag{1}$$

where a two-dimensional Laplace operator,

$$\nabla(\cdot) = \left(\frac{\partial(\cdot)}{\partial x}, \frac{\partial(\cdot)}{\partial y}\right), \tag{2}$$

and η is the groundwater table [m], b is the aquifer bed [m], S is the porosity [-], and $\gamma = I'L^2/k_0H^2$ is the normalized rainfall recharge where k_0 and I' are the aquifer's hydraulic conductivity [m s^{-1}] and the rate of rainfall [m s^{-1}], respectively. Equation (1) is verified to be equal to the classical solutions (Parlange et al., 1984; Chen and Liu, 1995).

3 Numerical scheme

To find numerical solutions, we used the leading-order equation of groundwater depth evolution, as below

$$\frac{\partial H}{\partial t} = \nabla \cdot H\nabla(H + b) + \gamma, \tag{3}$$

where $H = \eta - b$ is the total groundwater depth [m]. Equation (3) is a nonlinear advection-diffusion equation with a source term. To achieve efficient computation, an explicit scheme is adopted. With specific treatments for the nonlinear diffusion term and for assuring well-balancing property, a new Godunov-based relaxation scheme in the high resolution (LeVeque, 2002) is adopted to numerically solve (3).

4 Case study of different rainfall recharge

We consider an aquifer inclining at 30° with a constant hydraulic conductivity of $k_0 = 1.0 \times 10^{-3}$ and porosity of $S = 4.0$. An time variation of the peak rainrate value is assumed as is shown in Fig. 1. Four rainfall distributions are considered, including no rainfall, uniform, quadratic, and linearly decreasing distribution, as are shown in Fig. 2. We successfully obtain the hydrograph of volumetric discharge as well as the groundwater table of time variation.

4 Expected Results

A new model of shallow groundwater motion in an unconfined sloping aquifer with a spatial-varying rainfall recharge has been developed.

References

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LeVeque, R.J., *Finite Volume Methods for Hyperbolic Problems*, Cambridge University Press, 2002.

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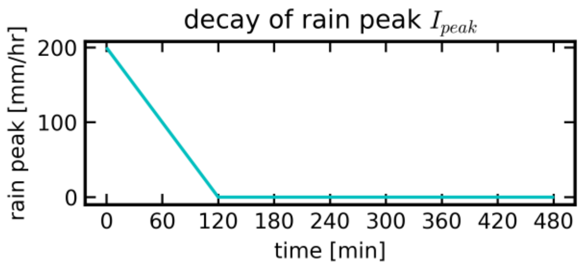

Fig. 1 Time variation of rainrate peak value

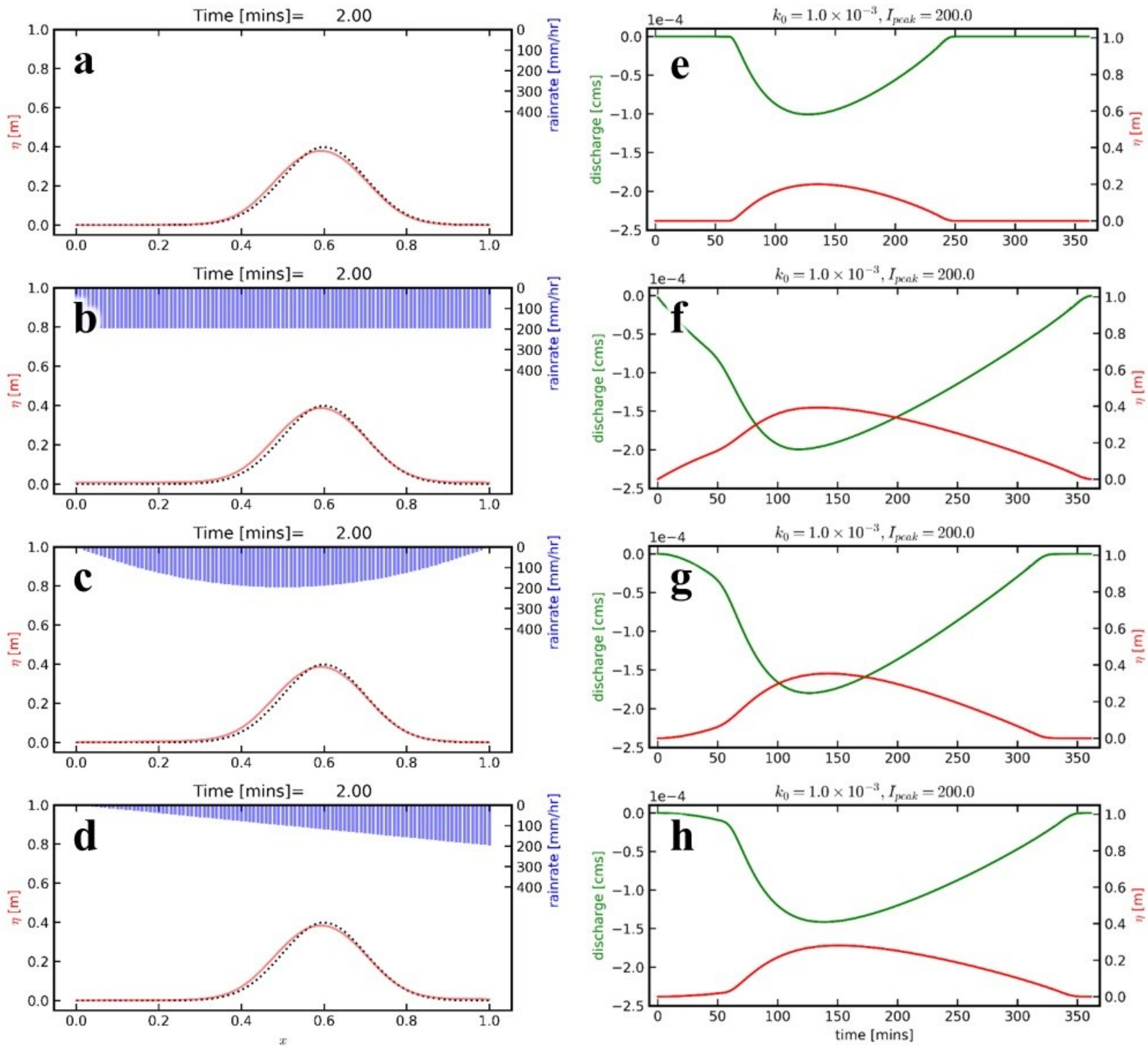

Fig. 2 The second-minute Groundwater table (red lines), the initial groundwater table (black dotted line) and rainfall patterns (blue bars) of (a) no rainfall, (b) uniform, (c) quadratic, and (d) linear distributions with a peak rainrate of 200 mm in the aquifer inclining at 30° and with a porosity $S = 4.0$ and of $k_0 = 1.0 \times 10^{-3}$. The corresponding outflow volumetric discharge (green lines) and downstream-boundary groundwater tables (red lines) of time variation under the (e) no rainfall, (f) uniform, (g) quadratic, and (h) linear rainfall distributions.