TWO-LAYER HYDROSTATIC FLOW MODEL IN A HORIZONTAL RECTANGULAR DUCT

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1. INTRODUCTION

Mixed flow phenomena occurs in many fields, namely; seawater intrusion into oil pipe mains, fluids intrusion in industrial facilities or water-air free surface flow in a closed conduit which resembles flow into underground spaces. (Hosoda, et. al,1992) investigated the water-air flow under the assumption of air and water incompressibility. HSMAC method was used for calculating pressure in the pressurized water region and air pressure was calculated separately. Their results of including air pressure in the downstream showed better agreement with experimental data. In this model we have further investigated the interaction between two incompressible fluids. HSMAC pressure calculations are applied to the entire domain. Figure (1) shows a schematic of the simulation problem and grid allocation for the variables. The simulation was conducted under the assumption of constant pressure in the upstream and incompressibility of both fluids.

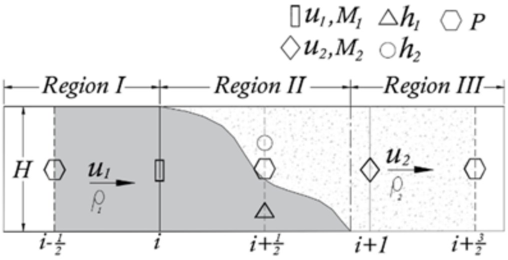

Fig.1 schematic for the flow regions and variables

Shown in figure.1 the main regions of the flow. Region (I) uniform pressurized flow of the upstream fluid; Region (II) mixed non uniform flow of the two fluids and (III) uniform flow of the downstream fluid.

2. GOVERNING EQUATIONS

The continuity and momentum equations for the upstream fluid –fluid (1) - are given by:

$$\frac{\partial h_1}{\partial t}+\frac{\partial M_1}{\partial x}=0 \tag{1}$$

$$\frac{\partial M_1}{\partial t}+\frac{\partial u_1 M_1}{\partial x}+\frac{\partial}{\partial x}\left(\frac{g{h_1}^2}{2}\right)+h_1\frac{\partial}{\partial x}\left(\frac{P+g\rho_2 h_2}{\rho_1}\right)=\tau_1 \tag{2}$$

where M_1 =upstream fluid discharge per unit width;$u_1 =$ upstream fluid depth averaged velocity; P= pressure atop duct; h_1= upstream fluid depth; g=gravitational acceleration;$\tau_{bx} =$ bottom shear stress; x=stream-wise distance; t=time; $\rho_1 =$ upstream fluid density; ρ_2 =downstream fluid density; H= duct height

The continuity and momentum equations for the downstream fluid –fluid (2)- are given by:

$$\begin{cases}\tau_1=\dfrac{-\tau_{bx}}{\rho_1}\ ;\ h_1<H\\ \tau_1=\dfrac{-2\tau_{bx}}{\rho_1}\ ;h_1=H\end{cases}\qquad \text{and} \qquad \begin{cases}\tau_2=\dfrac{-\tau_{bx}}{\rho_2}\ ;\ h_2<H\\ \tau_2=\dfrac{-2\tau_{bx}}{\rho_2}\ ;\ h_2=H\end{cases}$$

$$\frac{\partial h_2}{\partial t}+\frac{\partial M_2}{\partial x}=0 \tag{3}$$

$$\frac{\partial M_2}{\partial t}+\frac{\partial u_2 M_2}{\partial x}+\frac{\partial}{\partial x}\left(\frac{g{h_2}^2}{2}\right)+h_2\frac{\partial}{\partial x}\left(\frac{P}{\rho_2}\right)=\tau_2 \tag{4}$$

Where M_2 =downstream fluid discharge per unit width; u_2 =downstream fluid depth averaged velocity and h_2 = upstream fluid depth. Manning equation was used to calculate the bottom and ceiling shear as given by:

$$\frac{\tau_{bx}}{\rho}=\frac{gn^2u|u|}{h^{\frac{1}{3}}} \tag{5}$$

Considering the case of a duct of constant height H; adding Eq. 1 to Eq. 3 results

$$\frac{\partial M_1}{\partial x}+\frac{\partial M_2}{\partial x}=0 \quad \text{where} \quad \frac{\partial(h_2+h_1)}{\partial x}=\frac{\partial H}{\partial x}=0 \tag{6}$$

$$M_2+M_1=M_o \tag{7}$$

Where M_o= discharge at the upstream inlet.

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3. NUMERICAL METHOD

Finite volume method with staggered allocation of variables was used for the simulation, the discrete forms of continuity and momentum equations are given as follows. (INOUE,1986). For *j={1,2}*;

$$\frac{h_{j,i}^{n+1}-h_{j,i}^{n}}{\Delta t}+\frac{M_{j,i}^{n+1}-M_{j,i}^{n}}{\Delta x}=0 \tag{8}$$

$$\frac{M_{j,i}^{n+1}-M_{j,i}^{n}}{\Delta t}+\frac{u^{n}_{j,i+\frac{1}{2}}M_{j,i+\alpha}^{n+1}-u^{n}_{j,i-\frac{1}{2}}M_{j,i-1+\beta}^{n}}{\Delta x}+\frac{g}{2}\frac{(h^{n}_{j,i+\frac{1}{2}})^2-(h^{n}_{j,i-\frac{1}{2}})^2}{\Delta x}+\frac{1}{\rho_j}\frac{(h_{j,i+1/2}^{n}+h_{j,i-1/2}^{n})}{2}\frac{P_{i+1/2}^{n}-P_{i-1/2}^{n}}{\Delta x}+W=\tau_{j,i}^{n} \tag{9}$$

Where;

$$W=\begin{cases}\frac{g\rho_2}{\rho_1}\frac{h^{n}_{2i+\frac{1}{2}}-h^{n}_{2i-\frac{1}{2}}}{\Delta x} & ;j=1\,;\\ 0 & ;j=2\end{cases} \qquad \alpha=\begin{cases}1 & ;u_{i+1/2}^{n}\le 0\\ 0 & ;u_{i+1/2}^{n}>0\end{cases}\;;\qquad \beta=\begin{cases}1; & u_{i-1/2}^{n}\le 0\\ 0; & u_{i-1/2}^{n}>0\end{cases}\;;\qquad u_i^n=\begin{cases}M_i^n/h_{i-1/2}^n; & M_i^n>0\\ M_i^n/h_{i+1/2}^n; & M_i^n\le 0\end{cases}$$

$$u_{i+1/2}^{n}=\frac{u_{i+1}^{n}+u_{i}^{n}}{2} \tag{10}$$

HSMAC method. (Hirt and Cook, 1972) was used to correct the pressure in iterative manner as follows:

$$P_{i+1/2}^{*}=P_{i+1/2}^{n}+\delta P_{i+1/2}^{*} \tag{11}$$

$$\delta P^{*}_{i+\frac{1}{2}}=\frac{-2\epsilon^{*}\Delta x^{2}\rho_1}{\Delta t\left((h_1{}^{n}_{i-\frac{1}{2}}+2h_1{}^{n}_{i+\frac{1}{2}}+h_1{}^{n}_{i+\frac{3}{2}})+\frac{\rho_1}{\rho_2}(h_2{}^{n}_{i-\frac{1}{2}}+2h_2{}^{n}_{i+\frac{1}{2}}+h_2{}^{n}_{i+\frac{3}{2}})\right)} \tag{12}$$

The pressure $P_{i+1/2}^{*}$ is considered $P_{i+1/2}^{n+1}$ (n+1 is the next time step) when the criterion error$|\epsilon^{*}|\le 0.0001$, where ϵ^{*} is defined as:

$$\epsilon^{*}=\frac{(M_1+M_2)_{i+1}^{*}-(M_1+M_2)_{i}^{*}}{\Delta x} \tag{13}$$

4. RESULTS

Two types of simulation were conducted; (i) constant upstream pressure head with different density ratios and (ii) different upstream pressure heads with constant density ratio. Density ratiop $=\rho_2/\rho_1$ and pressure head $HP=P/g\rho_1$. Figures (2-5) show the results for pressure profiles and water profiles. Results show that interface region becomes larger as the downstream fluid density increases Fig.4. It also indicates similar behavior when the upstream pressure decreases Fig.5. The pressure profiles also show a linear pattern for the uniform flow regions; the ratio between pressure gradient in region (III) and region (I) is always equal to the ratio p $=\rho_2/\rho_1$that is independent from upstream pressure value as shown in figure (3) as a result of shear resistance ratio in both regions.

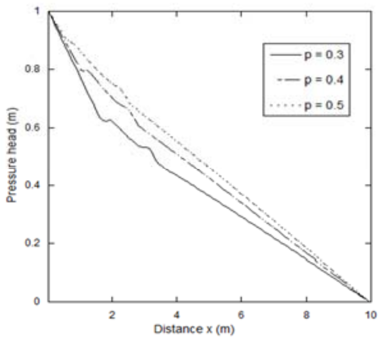

Figure.2 Pressure head at time = 1.3 sec for different p ratios

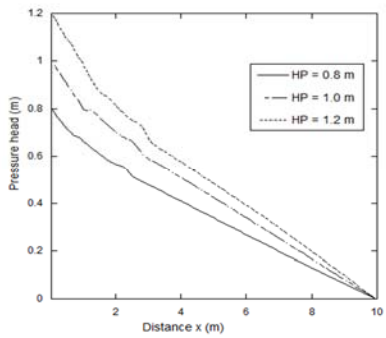

Figure.3 Pressure head at time = 1.3 sec different pressure heads

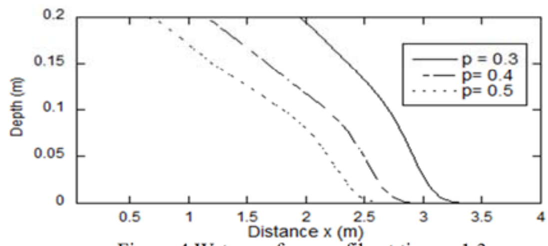

Figure.4 Water surface profile at time = 1.3 sec for different p ratios

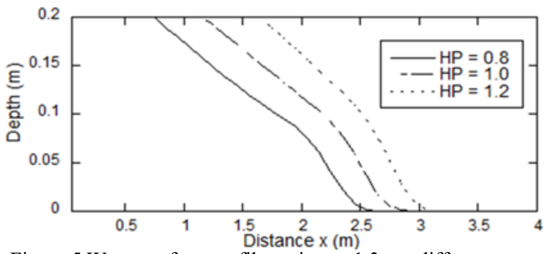

Figure.5 Water surface profile at time = 1.3 sec different pressure heads

4. CONCLUSION

The Numerical simulations show consistency with expected results of interface shape, extent and pressure profiles. Yet, model verification is required in order to fairly judge the results.

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