# A STUDY OF BRIDGE WASH-OUT SIMULATION DURING TSUNAMI BY USING THE SMOOTHED PARTICLE HYDRODYNAMICS (SPH) METHOD

Kyushu University Student Member OBodhinanda Chandra Kyushu University Regular Member Mitsuteru Asai

## **1. INTRODUCTION**

On March 11, 2011, a huge tsunami induced by the Great East Japan Earthquake devastated many public infrastructures in the pacific coast of northeastern Japan, particularly bridges. The collapse of bridges caused a serious problem of traffic disorder, contributing to the loss of life and leading to the delay of recovery correspondingly. After the tsunami, several disaster prevention and mitigation techniques are actively studied in order to develop better bridges and other coastal infrastructures; one of them is by utilizing computational mechanics simulation.

As bridges are essential for transporting people and goods in and out the disaster zone, further research to understand the bridge wash-out phenomena is considered important. The phenomenon is then selected as a target issue and represented by using a stabilized pure mesh-free numerical technique known as the Stabilized Incompressible SPH method [1], which has been actively developed within our research group recently. On top of that, a slip and no-slip boundary treatment technique by using virtual marker [2] is also implemented in order to obtain a more accurate, non-penetrating, and stable fluid flow simulation nearby wall and rigid body boundary. By utilizing those numerical treatments, we can evaluate much smoother pressure distribution in simulating the fluid-rigid body interaction behavior between the tsunami wave and the upper bridge structure during the wash-out phenomenon.

## 2. FLUID-RIGID BODY INTERACTION FORMULATION

#### 2.1 Fluid Dynamics

The governing equations of the incompressible fluid flow, the continuity and the Navier-Stokes equations, are represented by the following expressions

$$\nabla \cdot \boldsymbol{u} = 0 \qquad (1) \qquad \frac{D\boldsymbol{u}}{Dt} = -\frac{1}{\rho^0} \nabla p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{g} + \frac{1}{\rho^0} \nabla \cdot \boldsymbol{\tau} \qquad (2)$$

where u is the velocity field, p is the pressure, and v is the kinematic viscosity of the fluid, respectively. g and t indicate the gravitational acceleration and time while  $\rho^0$  is the constant fluid density as assumed in the general incompressible flow approach. The main objectives of the Incompressible SPH scheme is to solve the discretized pressure Poisson equation derived from the aforementioned governing equations at every time step to obtain an adequate pressure value.

#### 2.1 Rigid Body Dynamics

In this study, the general momentum conservation laws are solved numerically considering the external forces including the hydrodynamic forces as a fluid-structure interaction formulation. Upon receiving impacts from fluid and surrounding bodies, the total force is calculated considering the effect of gravity g, hydrodynamic forces at the rigid surface  $F_f$  and contact forces between the rigid body and fixed boundary  $F_e$ . These formulations are calculated by following equations.

$$\sum \mathbf{F} = M\mathbf{g} + \mathbf{F}_f + \mathbf{F}_e = M \frac{d\mathbf{T}}{dt} \qquad (3)$$
$$\mathbf{F}_f = \sum_i^{on \ the \ suface} P_i \Delta S_i \hat{\mathbf{n}}_i \qquad (4) \qquad \mathbf{F}_e = \mathbf{F}_n + \mathbf{F}_s \qquad (5)$$

where M is the total mass of the rigid body and  $P_i$  is the pressure on surface area  $\Delta S_i$  of rigid particle. In the current formulation, the contact force is decomposed into  $F_n$  and  $F_s$ , the normal and tangential components respectively. Both of these forces are modeled by using a penalty-based damped spring model inspired by the Hertz-Mindlin contact models in Discrete Element Modelling (DEM) formulation [3]. Once the hydrodynamic and external forces are obtained, the moment can be calculated subsequently by using the distance r between the target particle and center of gravity of the rigid body. The angular momentum conservation law is then expressed as

$$\sum \boldsymbol{M} = \boldsymbol{M}_{f} + \boldsymbol{M}_{e} = \boldsymbol{I} \frac{d\boldsymbol{\omega}}{dt} + (\boldsymbol{\omega} \times \boldsymbol{I}\boldsymbol{\omega})$$
(6)  
on the surface on the surface

$$\boldsymbol{M}_{f} = \sum_{i}^{on \ the \ surface} (\boldsymbol{r}_{i} - \boldsymbol{r}_{c}) \times \boldsymbol{F}_{f} \qquad (7) \qquad \boldsymbol{M}_{e} = \sum_{i}^{on \ the \ surface} (\boldsymbol{r}_{i} - \boldsymbol{r}_{c}) \times \boldsymbol{F}_{e} \qquad (8)$$

After obtaining the translational velocity T and the angular velocity  $\omega$ , the position and velocity of each rigid body particle can be updated accurately by using the knowledge of spatial variables of rigid body – rg and R. Vector rg is used

Keywords: Tsunami, Particle Simulation, SPH Method, Bridge Wash-Out

Contact address: Structural Analysis Lab., West 2-1102, 744 Motooka, Nishi-ku, Fukuoka, 819-0395, Japan, Tel: +81-92-802-3370

to describe the translation of the body's center and the 3x3 rotation matrix **R** is used to specify the rotation of the body about the center of mass. The position **x** and velocity **u** of each rigid body particle can be obtained by

$$\boldsymbol{x}_{i}^{n+1} = \boldsymbol{r}\boldsymbol{g}^{n+1} + \boldsymbol{R}^{n+1}\boldsymbol{r}_{i}^{0} \qquad (9) \qquad \boldsymbol{u}^{n+1} = \boldsymbol{T}^{n+1} + \boldsymbol{\omega}^{n+1} \times \boldsymbol{R}^{n+1}\boldsymbol{r}_{i}^{0} \qquad (10)$$

### 3. VERIFICATION AND VALIDATION TESTS

In order to validate our fluid-rigid body interaction formulations, a comparison between numerical solutions with experimental results is presented. The analysis model and the detail of the bridge girder model are shown in Fig. 1. In this experiment and analysis, a girder model was subjected to a dam-break flow upon the gate is opened.

The experimental tests were carried out several times in three different cases of initial water height *h*; 250mm, 300mm, and 350mm, and the location of the girder model is monitored by using a motion capture system. The bridge pier is fixed and the density of the girder is  $\rho = 1.161$  g/cm<sup>3</sup>, referring to the density of PLA filament, the material used to make the experimental model.

In the analysis, the particle diameter used is  $d_0 = 0.50$  cm, time increment  $\Delta t = 0.0005$ s and the total number of particles is about 2.4 million, 2.6 million, and 2.7 million for h = 250 mm, 300 mm, and 350 mm respectively. The flow is assumed to be 65% slip and the contact force parameters of the rigid body are;  $E = 5.0 \times 105 \text{ N/cm}^2$ , v = 0.28, and  $\varepsilon = 0.8$ . The frictional behavior between the girder and the pier and the glass wall are considered differently, with  $\mu_p = 0.05$  and  $\mu_w = 0.0$  respectively. Furthermore, the speed of opening gate is considered as the free fall velocity, similar to the load pulley gate opening mechanism which was used in the experimental tests.



Fig. 1. Detail of analysis model and motion capture system

Fig. 2 below shows the comparison of rotational angle between the experimental tests and the analysis for three different cases of initial water height. From these graphs, it can be clearly observed that our analysis results show a very good agreement with the experimental data, showing the rotation of girder from positive to negative angle. We believe that our proposed method can evaluate the fluid-rigid interaction motion during flow in a small scale experiment.



Fig.2. Comparison of rotational angle between experimental tests and analysis for initial water height; 250mm, 300mm, and 350mm, from left to right respectively

## 4. CONCLUSIONS

In this study, the verification and validation tests of fluid-structure interaction formulation were conducted after introducing rigid body dynamics algorithm considering penalty contact forces and spatial variables into ISPH. In our experimental validations, the rigid body motion shows a very good agreement compared to the experiment; particularly can be seen from the comparison of the rotational angle after wash-out. In the future works, consideration of the rigid body impact formulation will be studied and implemented before conducting a real scale bridge wash-out simulation. Further research should be conducted to improve of the numerical simulation in consideration of the detailed multibody systems of rigid and deformable bearings and aseismic connectors between the bridge upper and lower structures.

#### REFERENCES

- [1] Asai M., Aly A.M., Sonoda Y., and Sakai Y: A Stabilized Incompressible SPH method by relaxing the density invariance condition, Journal of Applied Mechanics, 24. doi:10.1155/2012/139583.
- [2] Asai M., Fujimoto K., Tanabe S., and Beppu M.: Slip and no-slip boundary treatment for particle simulation model with incompatible step-shaped boundaries by using a virtual maker, Transactions of JSCES, Paper No.20130011.
- [3] Di Renzo A. and Di Maio F.P.: Comparison of contact-force models for the simulation of collisions in DEM-based granular flow codes, Chemical Engineering Science, 59: 525-541.