STUDY ON IDENTIFICATION OF WAVE VELOCITY AND DENSITY OF DEFECT USING PARTICLE FILTER

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1. INTRODUCTION

Ultrasonic Testing (UT) has been performed to evaluate the safety of structural components. In the UT, the reflected signal from the defects are utilized for the estimation of their size and location. However, it is not easy to identify the structural interior parameter of the defect, such as the wave velocity, density, among others. To estimate the parameters from reflected signals, some kind of numerical approaches need to be introduced. The particle filter[1, 2] estimates the probabilistic density of the hidden state variables using observation variables. The observable variables are related to the hidden variables by some functional form that is known. In this research, the particle filter is applied into the identification of the wave velocity and density of a defect, which are hidden in the reflected signals. The relation between the state variables and reflected signals is described in the framework with finite integration technique[3] (FIT).

2. PARTICLE FILTER

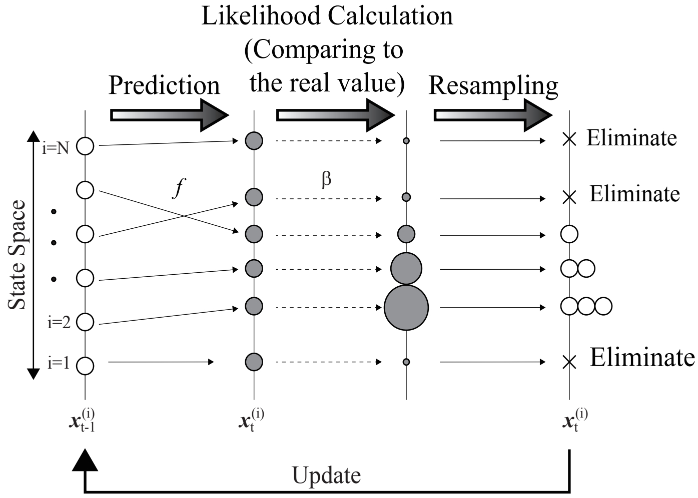

Fig. 1 Flowchart of Particle Filter

The particle filter aims to estimate the sequence of hidden parameters, $\boldsymbol{x}_t$ for $\boldsymbol{t} = 0, 1, 2, 3.......$, based only on the observed data $\boldsymbol{y}_t$. System equations with these properties are described as:

$$\boldsymbol{y}_t = h_t(\boldsymbol{x}_t^{(i)}) + \boldsymbol{w}_t^{(i)} \tag{1}$$

$$\boldsymbol{x}_t^{(i)} = f_t(\boldsymbol{x}_{t-1}^{(i)}) + \boldsymbol{v}_t^{(i)} \qquad \boldsymbol{v}_t \sim N(\mu, \sigma^2) \tag{2}$$

where the functions $\boldsymbol{h}$ and $\boldsymbol{f}$ are known function, and $\boldsymbol{w}_t$ and $\boldsymbol{v}_t$ are the measurement and the system noise, respectively. The system noise distributes with the average μ and variance σ^2. Here we may have N samples(particles) to show the probabilistic density of $\boldsymbol{x}_t$, and the samples are labeled with superscripts as $\boldsymbol{x}_t^{(1)}$,$\boldsymbol{x}_t^{(2)}$, ..., $\boldsymbol{x}_t^{(N)}$. At each step, the importance weight (the likelihood) of the particle is calculated as:

$$w_t^{(i)} = \frac{\exp\left\{-\frac{1}{2}\left(\boldsymbol{y}_t - h_t(\boldsymbol{x}_t^{(i)})\right)^T R_t^{-1}\left(\boldsymbol{y}_t - h_t(\boldsymbol{x}_t^{(i)})\right)\right\}}{\sqrt{(2\pi)^m|\boldsymbol{R}_t|}} \tag{3}$$

where $\boldsymbol{R}$ is the variance-covariance matrices. The step of the particle filter method in this study is as follow:

A. $\boldsymbol{x}_0^{(i)}$: Initial variables are randomly generated.

B. For $t = 1, \cdots, T$, step (a), (b), (c) are calculated.

   (a) For each particle i, perform the following steps:

      i. Generate random variable: $\boldsymbol{x}_t^{(i)}$

      ii. Prediction: $\boldsymbol{x}_t^{(i)} = f_t(\boldsymbol{x}_{t-1}^{(i)}) + \boldsymbol{v}_t^{(i)}$

      iii. Likelihood Calculation: $w_t^{(i)}$

   (b) Compute the weight summation $\sum_{j=1}^{N} w_t^{(j)}$

   (c) For resampling and updating of particles, $\boldsymbol{x}_t^{(i)}$ is evaluated using:
   $\beta^{(i)} = \frac{w_t^{(i)}}{\sum_{j=1}^{N} w_t^{(j)}}$

The above procedures are repeated, and the state vectors are continually updated. This method is called the Sequential Importance Resampling (SIR). As shown in

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Fig.1, the particles with low likelihood are eliminated, and those with high likelihood are remained or divided for the generation of new particles.

3. FINITE INTEGRATION TECHNIQUE (FIT)

The FIT is a method for the simulation of elastic wave propagation and scattering. Since the FIT is discretized in a unified grid form, an image-based modeling can be applied. In the image-based modeling, a numerical model can be made from digital images, and then pixel or voxel data is directly fed into the FIT simulation. We have already proposed a parallel calculation technique using general-purpose computing on GPUs. In this research, the 2D FIT accelerated with GPU calculation is utilized to model the functional form $\boldsymbol{f}$ in Eq.(1).

4. IDENTIFICATION OF WAVE VELOCITY AND DENSITY IN AN INCLUSION

Here we consider the problem for identification of the SH-wave velocity c_T and the density ρ in an inclusion. The state variables are shown as:

$$\boldsymbol{x} = [c_T, \rho]^T \tag{4}$$

In this problem, the state variables are independent of the time. Therefore, the posterior state can be expressed using the prior state as:

$$\boldsymbol{x}_t = \boldsymbol{x}_{t-1} + \boldsymbol{v}_t \tag{5}$$

Here we demonstrate the identification of the wave velocity c_T and density ρ in the inclusion using the particle filter. As shown in Fig.2, ultrasonic SH wave with 1 MHz frequency is transmitted into an elastic material from a transducer with 10 mm diameter. The Fourier spectrum of the reflected wave from the inclusion is used as the measured variables. Here we substitute the numerically calculated spectrum for the measured one. Although the particle filter uses large number of particles in general, 10 particles are utilized considering the simplicity of the problem. The true value of c_T and ρ are 2400 $\mathrm{m/s}$ and 7000 $\mathrm{kg/m^3}$, respectively. In this study, the initial value of c_T are distributed between 1000 $\mathrm{m/s}$ and 3000 $\mathrm{m/s}$, and the one of the ρ are between 4000 $\mathrm{kg/m^3}$ and 8500 $\mathrm{kg/m^3}$. The allocation of the particle after 30 steps is shown in Fig.3. It is found that the particles converge on the true value.

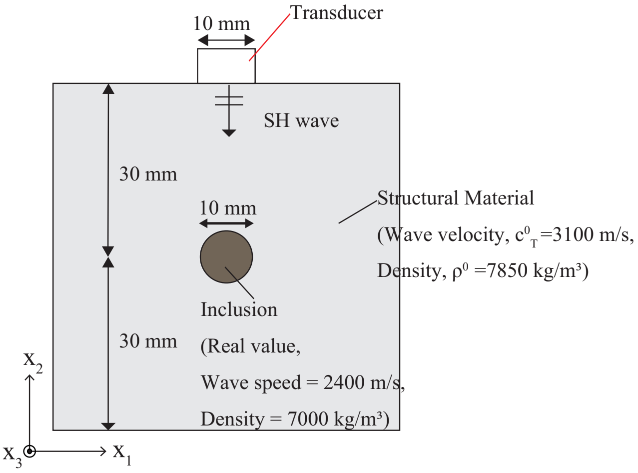

Fig. 2 An inclusion embedded in matrix and ultrasonic SH wave with 1MHz frequency.

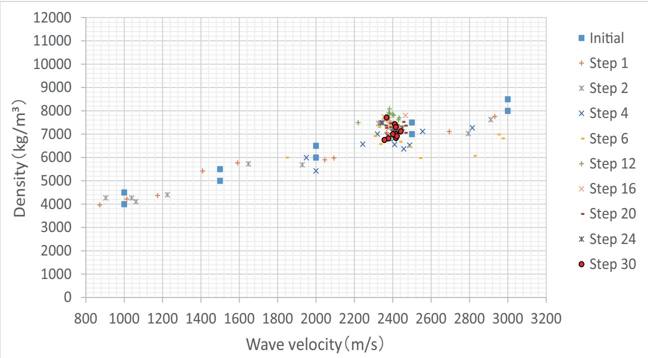

Fig. 3 Estimation result with the Particle Filter

5. CONCLUSIONS

For the UT, the particle filter was applied for estimation of the structural parameters of an inclusion. In the simulation model, the finite integration technique was introduced for the calculation of the reflected ultrasonic signal from the inclusion. The demonstration showed the good estimation result of the wave velocity and density. In the future research, we will validate our approach using experimentally measured signals.

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