STATIC STRESS FIELDS IN LOOSE SAND HEAPS LOADED BY SELF WEIGHT

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1. INTRODUCTION

The authors have already gained the achievement in both experimental and analytical study of self-weight distribution in loose sand heap whose side boundaries of the wedge are symmetrically inclined at the angle of repose (Pipatpongsa et al., 2010, 2014). The derived stress distributions show a good agreement with past experimental studies and measured results obtained from sand heap models under 1g condition. Based on the assumptions used in our past studies, the characteristic of the problem is self-similar and have no inherent length scale. Therefore, stress distributions must appear in the similar geometric shape at all distances from the apex of sand heap. Despite a boundary between sand and a rigid flat base causes an abrupt change in boundary condition, these local effects would not be likely to affect the stresses in the region of interest because the boundary influence is asymptotically decay with distance in elastic body by virtue of Saint-Venant's principle. The self-similarity can be achieved with its immediate consequence that the stress solutions can be expressed in the separated-variable form between its depth and gradient. The present study shows that the further attractive attribute of self-similarity of sand heap is the analytical method which can calculate profiles of shear stress and horizontal stress once a profile of vertical stress is given by using static equilibrium requirement and boundary conditions. That means a vertical stresses distribution measured by pressure gauges on the base could be used to estimate shear stress and horizontal stress distributions.

2. THEORETICAL BACKGROUND

Three in-plane stress components which are horizontal stress σ_x, vertical stress σ_z and shear stress τ_{xz} must satisfy the following two-dimensional equilibrium condition under self-weight loading with constant unit weight γ where x and z represent horizontal distance from the ridge of the wedge and vertical distance along the gravity, respectively.

$$\partial\sigma_x/\partial x+\partial\tau_{xz}/\partial z=0,\quad \partial\tau_{xz}/\partial x+\partial\sigma_z/\partial z=\gamma \qquad (1),(2)$$

As a consequence of self-similarity of the problem, stress components are distributed in the similar geometric shape at all distances from the ridge of sand heap; therefore, each of them can be expressed in term of scaling stresses χ_x, χ_z and χ_{xz} where s represents a relative gradient of the slope with a range between 0 to 1, and ϕ is an angle of repose.

$$\sigma_x(x,z)=\gamma z\chi_x(s),\quad \sigma_z(x,z)=\gamma z\chi_z(s),\quad \tau_{xz}(x,z)=\gamma z\chi_{xz}(s),\quad s=x\tan\phi/z \qquad (3),(4),(5),(6)$$

Partial derivatives shown in Eq.(1) and Eq.(2) are replaced by the following differential equations where $\chi'_x=d\chi_x/ds$, $\chi'_z=d\chi_z/ds$ and $\chi'_{xz}=d\chi_{xz}/ds$, using the chain rule differentiation

$$\partial\sigma_x/\partial x=\gamma\chi'_x\tan\phi,\quad \partial\sigma_z/\partial z=\gamma\left(\chi_z-s\chi'_z\right),\quad \partial\tau_{xz}/\partial x=\gamma\chi'_{xz}\tan\phi,\quad \partial\tau_{xz}/\partial z=\gamma\left(\chi_{xz}-s\chi'_{xz}\right) \qquad (7),(8),(9),(10)$$

According to Eqs.(3)–(10), Eq.(1) and Eq.(2) are arranged to the following equations.

$$\chi'_x=\left(s\chi'_{xz}-\chi_{xz}\right)\cot\phi,\quad \chi'_{xz}=\left(1+s\chi'_z-\chi_z\right)\cot\phi \qquad (11),(12)$$

Obviously, Eq.(11) and Eq.(12) imply that if the scaling stress χ_z is known; χ_{xz} and χ_x can be theoretically obtained by integrating the derivatives with stress-free condition at the sliding plane where s=1.

$$\chi_x\left(1\right)=0,\quad \chi_z\left(1\right)=0,\quad \chi_{xz}\left(1\right)=0 \qquad (13),(14),(15)$$

Integrations of Eq.(11) and Eq.(12), considering Eqs.(13)–(15), can be achieved in terms of definite integration.

$$\chi_x=\left(s\chi_{xz}-2\int_1^s\chi_{xz}ds\right)\cot\phi,\quad \chi_{xz}=\left(s\chi_z+\int_1^s 1-2\chi_z ds\right)\cot\phi \qquad (16),(17)$$

Though all stresses vanish at the sliding plane, the incipient failure condition of infinite slope defines Eq.(18) and Eq.(19) where the Mohr-Coulomb failure criterion as expressed in Eq.(20) is entirely yielded with constant principal stress direction.

$$\sigma_z=\sigma_x+2\tau_{xz}\tan\phi,\quad \chi_z=\chi_x+2\chi_{xz}\tan\phi,\quad Y=\sqrt{\left(\tfrac{1}{2}\left(\sigma_x-\sigma_z\right)\right)^2+\tau_{xz}^2}-\tfrac{1}{2}\left(\sigma_x+\sigma_z\right)\sin\phi \qquad (18),(19),(20)$$

Substitutions of Eqs.(13)–(15) and Eq.(19) into Eq.(11) and Eq.(12) obtain the derivatives of scaling stress components with respect to s at the sliding plane where s=1.

$$\chi'_x\left(1\right)=-\cos^2\phi,\quad \chi'_z\left(1\right)=-\left(1+\sin^2\phi\right),\quad \chi'_{xz}\left(1\right)=-\sin\phi\cos\phi \qquad (21),(22),(23)$$

The vertical stress at the central plane of sand heap where s=0 with a hump profile is characterized by Eq.(24). The horizontal-to-vertical stress ratio at the central plane passing the ridge is given as a parameter K in Eq.(25).

$$\chi'_z\left(0\right)=0,\quad \chi_x(0)/\chi_z(0)=K \qquad (24),(25)$$

Keywords: stress analysis, angle of repose, load transmission, statics, embankment
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3. METHOD OF ESTIMATION

According to Booth (1938), a fourth-order polynomial equation is sufficient to describe the profile of scaling vertical stress χ_z as a function of a slope gradient s. The estimated function is given below where a_4, a_3, a_2 and a_1 are unknown coefficients of polynomial satisfying $\chi_z(1)=0$ (see Eq.(14)).

$$\chi_z(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s - \left(a_4 + a_3 + a_2 + a_1\right) \tag{26}$$

Substituting Eq.(26) into Eq.(16) obtains the corresponding functions of χ_x as expressed in Eq.(27). Similarly, the corresponding functions of χ_{xz} is obtained in Eq.(28).

$$\chi_x(s) = \left(\tfrac{2}{5}a_4\left(s^6-1\right) + \tfrac{3}{10}a_3\left(s^5-1\right) + \tfrac{1}{6}a_2\left(s^4-1\right)\right)\cot^2\phi \tag{27}$$

$$\chi_{xz}(s) = \left(a_4\left(\tfrac{3}{5}s^5+1\right) + a_3\left(\tfrac{1}{2}s^4+1\right) + a_2\left(\tfrac{1}{3}s^3+1\right) + a_1 + 1\right)s\cot\phi \tag{28}$$

Booth (1938) assumed additional conditions on the normalized weight of half-width heap as given in Eq.(29). Thus, integration of vertical stress of sand heap along an half-width heap b with a height h and a bulk unit weight γ equals to a half-volume of sand heap times bulk unit weight as given in Eq.(30).

$$\int_0^1 \chi_z(s)ds = \frac{1}{2}, \quad \int_0^b \sigma_z(x,h)dx = \frac{1}{2}(\gamma h)_{\text{fitting}}\, b \tag{29),(30}$$

According to Booth (1938), the solutions of a_4, a_3, a_2 and a_1 are expressed below, using Eq.(22), Eq.(24) and Eq.(29) where J, E, H are given by Eqs.(35)–(37). Note that active limit was assumed in Booth (1938) by assigning K=(1-sinϕ)/(1+sinϕ); however, K is considered as an unknown parameter to be fitted with experimental data in this study.

$$a_4 = \tfrac{5}{2} - \tfrac{15}{4}J + 25H, \quad a_3 = \tfrac{-10}{3} + \tfrac{20}{3}J - \tfrac{160}{3}H, \quad a_2 = -3J + 30H, \quad a_1 = 0 \tag{31),(32),(33),(34}$$

$$J = 1 + \sin^2\phi, \quad E = \left(\tfrac{5}{6} + \tfrac{1}{12}J\right)\Big/\left(1 + \tfrac{5}{3}K\tan^2\phi\right), \quad H = KE\tan^2\phi \tag{35),(36),(37}$$

4. COMPARISON WITH EXPERIMENT

Pipatpongsa et al. (2014) reported 5 sets of experimental data of vertical pressure acting along the base of the wedge-shaped sand heap built by three different sedimentations of silica sand no.8. Angle of shearing resistance is ϕ_{DS}=35º, minimum and maximum unit weights are γ_{min}=11.43 kN/m^2 and γ_{max}=15.00 kN/m^2. Half-width base b=150 mm, average angle of repose ϕ=36º, average height h_{avg}=110 mm. Parameters K and $(\gamma h)_{fitting}$ are fitted from the experimental data using Eq.(26), giving K=0.253 and $(\gamma h)_{fitting}$=1.478 kN/m^2; hence $\gamma_{fitting}=(\gamma h)_{fitting}/h_{avg}$=13.44 kN/m^3. Fig.1 shows stress components calculated from the theory (Pipatpongsa et al., 2010) and the fitting method in comparison with experimental data. Fig.2 indicates that stress components calculated from the fitting method violates the yield function.

5. CONCLUSION

Though the fitting stress field using polynomial function is not admissible static stress fields, static stress components and bulk unit weight can be estimated from vertical stresses measured at the base of sand heap.

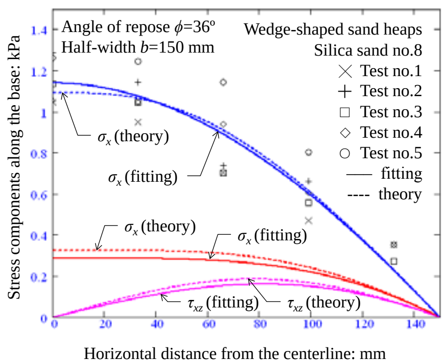

Fig. 1 Comparison between profiles of stress components and experimental vertical stress acting along a half-width base of sand heaps

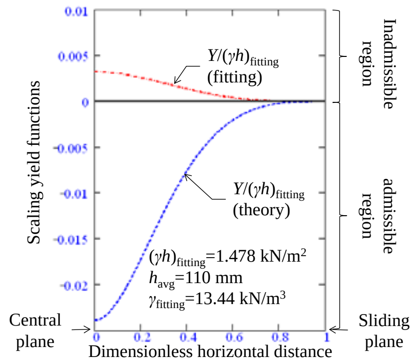

Fig. 2 Distribution of scaling yield functions along dimensional horizontal distance

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