MODEL UPDATING OF A STEEL TRUSS BRIDGE USING BAYESIAN PROBABILISTIC APPROACH

Saitama University Student Member ○Samim Mustafa
Saitama University Regular Member Yasunao Matsumoto

1. INTRODUCTION

Numerical models are usually used to predict the actual behaviour of real system in almost all the fields of engineering. For example, finite element (FE) models are used in civil and structural engineering for structural analysis, structural control and for structural health monitoring (SHM). However, such numerical models are constructed on the basis of highly idealized engineering design and knowledge that may detract from the quality and accuracy of these models and its purposes. As a result, a significant discrepancy may exist between the predicted FE model and actual built structure. In order to minimize this discrepancy, effective methods for FE model updating are needed to calibrate the initial FE model by using experimental data obtained from actual tested structure.

In this paper, an efficient and robust Bayesian probabilistic approach is presented for finite element model updating utilizing vibration data measured in an existing steel truss bridge. The proposed algorithm is validated experimentally to illustrate the efficiency of the method.

2. BAYESIAN MODEL UPDATING

2.1 The basic framework

The basic framework of the method proposed is based on Bayes' theorem. Let D denote available data from a dynamic system under consideration. By using Bayes' theorem, the updated or posterior probability of the model parameter χ given the available data and a model class C is given by Vanik et al. (2000):

$$p(\chi \mid D,C) = \frac{p(D \mid \chi,C)\, p(\chi \mid C)}{p(D \mid C)} \tag{1}$$

2.2 Parameterization

A parameterization for the stiffness and mass matrices for a linear structural model with N_d degrees of freedom is:

$$K(\theta) = K_0 + \sum_{i=1}^{N_\theta} \theta_i K_i \quad \text{and} \quad M(\vartheta) = M_0 + \sum_{j=1}^{N_\vartheta} \vartheta_j M_j \tag{2}$$

where K_0, M_0, K_i and M_j all are constant matrices independent of the model parameters and $\theta_i\left(i=1,\ldots,N_\theta\right)$; $\vartheta_j\left(j=1,\ldots,N_\vartheta\right)$ are the stiffness and mass parameters respectively which need to be updated to make necessary adjustment in the FE model in order to make it more consistent with the real structure.

2.3 Objective function

Substituting the expression for likelihood function and prior PDF into Eq. (1), the expression for the posterior PDF can be obtained. For the ease of this optimization problem, the negative logarithm of posterior PDF was taken as the objective function which is given by:

$$\Gamma(\theta,\vartheta,\lambda,\phi) = \frac{1}{2}(\theta-\theta_n)^T \Sigma_\theta^{-1}(\theta-\theta_n) + \frac{1}{2}(\vartheta-\vartheta_n)^T \Sigma_\vartheta^{-1}(\vartheta-\vartheta_n) + \frac{1}{2\sigma_e^2}\sum_{r=1}^{N_m}\left\|\left(K(\theta)-\lambda_r M(\vartheta)\right)\phi_r\right\|^2 + \frac{1}{2}\sum_{r=1}^{N_m}\frac{1}{\sigma_{f,r}^2}(\lambda_r-\hat{\lambda}_r)^2 + \frac{1}{2}\sum_{r=1}^{N_m}\frac{1}{\sigma_{s,r}^2}\left(\frac{\hat{\psi}_r^T L\phi_r}{\left|\hat{\psi}_r\right|\left|L\phi_r\right|}-1\right)^2 \tag{3}$$

The above objective function was formulated by excluding the constant that did not depend on the uncertain model parameters. The most probable values (MPV) of unknown parameters were obtained by minimizing this objective function sequentially and iteratively until some prescribed convergence criteria was satisfied (Yuen et al. (2006)).

3. APPLICATION

3.1 Vibration measurements and system identification

The bridge studied was a warren type steel truss bridge consisting of five main spans, each having length of 70.77 meters (Yamaguchi et al. (2015)). Due to its geometric similarity, only one span (approach span) was modelled in MATLAB. The structural members were modelled as three-dimensional frame elements. Six DOF were considered at each joint. The employed numerical model was a 2510 DOF FE model consisting of 514 frame elements and 420 nodes with 10 boundary conditions. The vibration measurement was conducted by a car running test on the bridge using piezoelectric accelerometers and servo velocity-meters as shown in Fig. 1(a). The data were collected for 10 min at a sampling rate of 100 Hz. Stochastic subspace identification (SSI) was used to identify the modal properties from free vibration responses

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Contact address: 255, Shimo-okubo, Sakura-ku, Saitama, 338-8570, Japan, Tel: +81-48-858-3557

extracted from full length recorded data by checking the traffic free time on the bridge from the video of traffic recorded during measurement. Five bending modes and two torsional modes were identified, as shown in Fig. 1(b).

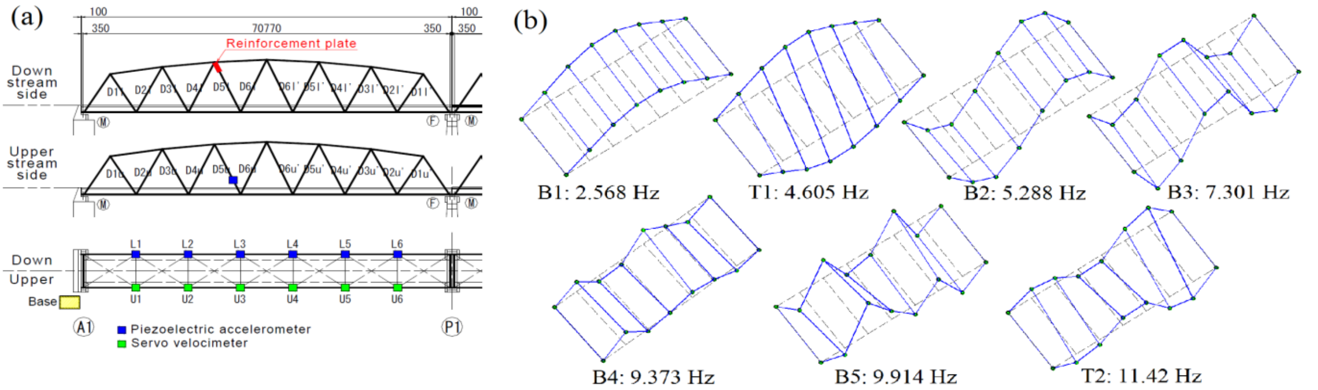

Fig. 1 (a) Layout of sensor location; (b) Identified modes by SSI; B: Bending mode, T: Torsional mode

3.2 Model updating results

The model updating was carried out using proposed algorithm considering 1% coefficient of variations of the measurement error in the squared modal frequencies and mode shapes respectively for all modes. Table 1 shows the model updating results for the modal frequencies and MAC values. It can be seen that the frequencies of the updated model are very close to the measured ones and MAC values are also improved, validating the proposed algorithm. Fig. 2 shows the comparison between the measured mode shapes (red lines) and the updated system mode shapes (green lines). The results show that the two sets of curves are almost top of each other which also suggests that the updating method is efficient in updating mode shapes though only limited components of modes were measured.

Table 1. Identification results for frequencies and MAC values

Experimental order of Mode	Measured f(Hz)	Initial f(Hz)	Updated f(Hz)	MAC initial	MAC Updated
1st Bending	2.569	2.599	2.569	0.9956	0.9998
1st Torsional	4.605	4.746	4.605	0.9987	0.9997
2nd Bending	5.288	5.444	5.288	0.9917	0.9986
3rd Bending	7.302	7.722	7.306	0.9502	0.9998
4th Bending	9.376	9.311	9.384	0.7651	0.9983
5th Bending	9.915	9.974	9.918	0.7599	0.9978
2nd Torsional	11.42	11.66	11.43	0.8743	0.9998

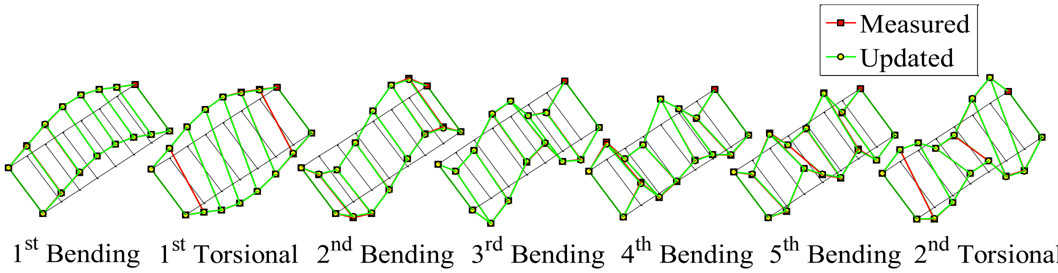

Fig. 2 Comparison between actual mode shapes and updated system mode shapes

4. CONCLUSIONS

In this paper an initial MATLAB-based FE model was updated using the identified modal properties by the proposed framework. To get the most probable values of the model parameters, an efficient iterative algorithm was used, which involves a series of coupled linear optimization problems. The experimental study confirms the effectiveness of the proposed approach, showing it to be both computationally efficient and robust. This makes the propose method a deserving candidate for model updating of a large-scale structure with incomplete measured modal data.

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