BRIDGE NATURAL FREQUENCY ESTIMATION BY EXTRACTING THE COMMON VIBRATION COMPONENT FROM THE RESPONSES OF TWO VEHICLES

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1. INTRODUCTION

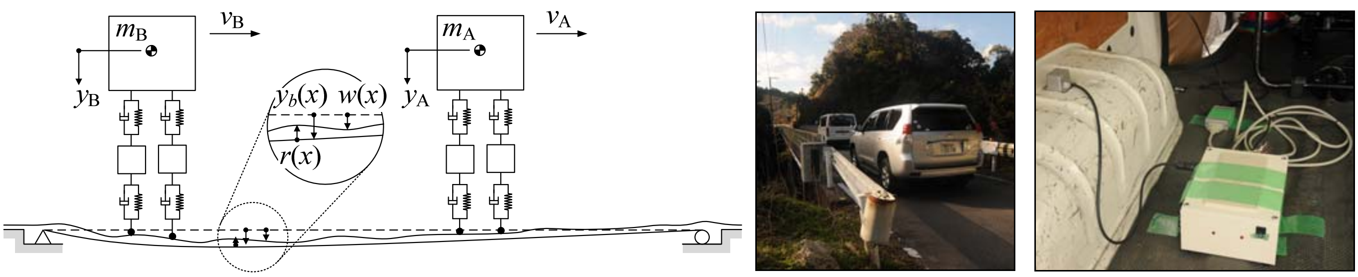
The measurement of the bridge natural frequency is widely practiced in bridge engineering. Traditionally, such measurement involves direct installation of instruments to the structure in question, a process that is costly and often encounters site-specific technical difficulties. For this reason, application of such measurement strategy to a large stock of bridges is cumbersome. Studies have investigated the possibility of using vehicle to obtain the natural frequency, such as those conducted by Yang et al. (2004a) McGetrick et al. (2009) and Siringoringo et al. (2012). Vehicle-based measurement centers on the concept that bridge vibration influences the vehicle response as a result of vehicle-bridge interaction (VBI) process. Therefore it is theoretically feasible to obtain bridge natural frequency from the frequency spectrum of the vehicle response. In practice, however, application of vehicle-based measurement is met by difficulties. Typically, spectral peak associated with bridge frequency is low in magnitude. In some cases, road roughness components are prevalent, to the extent that the random frequency peaks nullify the desirable bridge related frequency peak. Upon computation of the frequency spectrum, engineers are expected to conduct 'blind assessment' without any *a priori* knowledge of the exact bridge frequency. The presence of random frequency peaks combined with a weak bridge frequency peak (or a complete lack thereof) potentially leads to a misleading frequency identification.

2. EXTRACTION OF BRIDGE NATURAL FREUQUENCY USING CROSS-SPECTRUM

A vehicle on a bridge is excited simultaneously by two external influences, i.e., bridge vibration and road roughness. This is true under the assumption of near-zero active external forces that are acting upon the vehicle. Ideally, the spectra of the responses of two vehicles contain the common frequency components from the bridge vibration and road roughness. For the latter, due to its largely random nature and variation in driving speeds, typically the vehicle response associated to it is blurred; thus less common across different spectra. Cross-spectrum is a simple yet powerful tool to identify common frequency components within two different random processes:

$$G_{xy}(f) = \lim_{T\to\infty} \frac{2}{T} E\left[X_k^*(f) Y_k(f) \right] \tag{1}$$

Figure 1a describes the measurement strategy that employs a cross-spectrum computation process. Under this strategy, two vehicles are employed to simultaneously capture the vertical acceleration responses, i.e., $\ddot{y}_A$ and $\ddot{y}_B$ from vehicle A and B, respectively. Since vehicles are running in a consecutive manner, both vehicles are expected to experience identical bridge responses. On the other hand, road roughness profiles experienced by both vehicles are expected to be less identical, due to variation in running trajectories and driving speeds.



(a) FEM model of measurement strategy *(b) Measurement* *(c) Vehicle instrumentation*

Figure 1 Measurement strategy using two vehicles

3. NUMERICAL FEASIBILITY STUDY

3.1 Vehicle-Bridge Interaction (VBI) Formulation

Several Vehicle-Bridge Interaction system methods have been developed with acceptable resulting accuracy. Efficiency in term of computational resource usage, nonetheless, varies. Element-level coupling method, as pointed out by Yang et al. (2004b) is found to have the best performance in term of computational resource:

$$\left[\mathbf{M}_{\mathrm{v}}\right]\left\{\ddot{\mathbf{u}}_{\mathrm{v}}\right\}+\left[\mathbf{C}_{\mathrm{v}}\right]\left\{\dot{\mathbf{u}}_{\mathrm{v}}\right\}+\left[\mathbf{K}_{\mathrm{v}}\right]\left\{\mathbf{u}_{\mathrm{v}}\right\}=\left\{\mathbf{F}_{\mathrm{cv}}\right\};\ \left[\mathbf{M}_{\mathbf{b},i}\right]\left\{\ddot{\mathbf{u}}_{\mathbf{b},i}\right\}+\left[\mathbf{C}_{\mathbf{b},i}\right]\left\{\dot{\mathbf{u}}_{\mathbf{b},i}\right\}+\left[\mathbf{K}_{\mathbf{b},i}\right]\left\{\mathbf{u}_{\mathbf{b},i}\right\}=\left\{\mathbf{F}_{\mathbf{cb},i}\right\} \tag{2}$$

Equation 1 and 2 represent the equation of motions associated to vehicle and bridge, respectively. Note that the index i denotes the bridge element upon which vehicle is interacting. The index emphasizes the feature of element-level coupling where temporal manipulation of bridge is carried out element-wise.

3.2 Numerical results

Keywords: Vibration measurement, Bridge natural frequency, Vehicle-bridge interaction, Synchronized sensing
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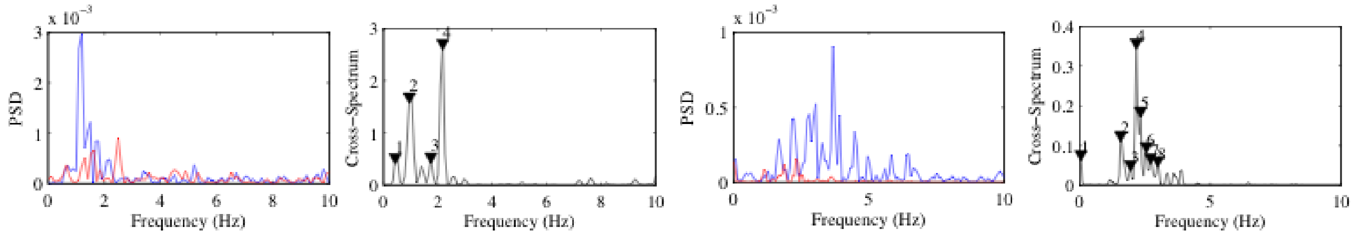
Figure 2a presents PSDs and Cross-spectrum of a simulated realization of two vehicles passing a bridge successively with 30 km/h driving speed. One can see that four prominent peaks have appeared in the computed cross-spectrum. The prominence of a peak is defined by the *clarity index*, with the following formulation:

$$I_i = \frac{G(\omega_i)}{\frac{1}{n}\sum_{\omega=\omega_l}^{\omega_u} G(\omega)} \tag{3}$$

where ω_i is the frequency value of interest, ω_l and ω_u are the lower and upper frequency limit of the displayed frequency interval assigned as 0 and 10 Hz, respectively. A peak is considered as “prominent” when its associated carity index is greater than 3. At this stage, it is not possible to identify a peak which is a bridge natural frequency without *a priori* knowledge of the value. One way to identify whether a peak is associated to the bridge dynamic characteristic or not, would be to compute the relative change of clarity indices ΔI_i for all prominent peaks between those of cross-spectrum and individual spectra (spectrum vehicle A and B). Table 1a presents ΔI_i for i = 1, 2, 3, 4. It is apparent that peak #4, at 2.18 Hz, exhibits the largest increase in clarity index I_i ; a strong indication of frequency commonality. From the eigensolution of the FEM model of the bridge the first natural frequency is obtained as 2.21 Hz, very close to peak #4.



(a) Numerical *(b) Experimental*

Figure 2 PSD and Cross-spectrum (— PSD A, — PSD B, and — cross-spectrum)

4. EXPERIMENTAL STUDY

An experimental study has been conducted in 2014 with a 59 m simply supported steel box-girder bridge as the test bed. The target bridge, Tsukige Bridge, is situated in Kimitsu City, Chiba Prefecture, Japan. A dynamic test has been conducted in 2009, from which bridge natural frequency is obtained as 2.14 Hz. Figure 1b, c illustrate the measurement process in the field. High accuracy MEMS-type accelerometers are installed on the two vehicles and are synchronized with each other using the GPS signals. The synchronization accuracy is estimated as better than 1 ms. Figure 2b presents PSDs and Cross-spectrum of the field measurement. Table 1b presents the corresponding ΔI_i for i = 1, 2,…, 7, 8. Peak #4 at 2.17 Hz exhibits the largest increase in clarity index I_i . The test was repeated four times with different drive speed combinations; the natural frequency was identified in all these cases.

Table 1 Clarity index changes ΔI_i.

(a) Numerical

Vehicle	Peak #			
	1	2	3	**4**
A	3.93	5.70	-0.11	○ **17.94**
B	4.01	12.30	0.76	○ **16.60**

(b) Experimental

Vehicle	Peak #						
	1	....	3	**4**	5	....	8
A	3.70	....	0.66	○ **20.28**	9.50	....	-1.25
B	5.36	....	-4.26	○ **13.55**	-0.72	....	2.73

‘○’ indicates a peak with strong frequency commonality.

5. CONCLUDING REMARKS

This paper proposes the strategy to indirectly estimate the bridge natural frequency using vibration responses of two vehicles. From the numerical study, the strategy is verified to be feasible. Similar result is obtained from the field measurement, confirming the practicality of the strategy.

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