An analytical model for one dimensional, transient evaporation in homogeneous soil

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1. INTRODUCTION

Evaluating evaporation process from soil surface is of great importance for many practical problems in geotechnical and geoenvironmental engineering. Soil evaporation process is influenced by various factors, such as soil texture, initial water content, hydraulic conductivity, water retention ability, water table level, etc. These factors don't function as independent variables, but rather act as a closely coupled system. Because of these involvements, a comprehensive demonstration of evaporation mechanism is still in lack.

Several researchers have presented analytical or quasi-analytical solutions of Richards equation for soil evaporation process in presence or absence of water table, it is not sufficiently enough to clarify the evaporation mechanism. Reasons are because these studies were mainly focused on an initial uniform soil profile. Furthermore, evaporative fluxes were usually treated as a constant or treated as variable with basic function, whereas rare analytical work responded the true evaporation curve. Moreover, a comparison between theory and field or laboratory observation was lack, as a result, it is hard to determine the accuracy of these models. This study aims to overcome the imperfections mentioned above, and present an analytical model for modelling water content redistribution during evaporation process.

2. THEORY

Richards equation is a general partial differential equation describing water flow in unsaturated, non-swelling soils. By invoking Darcy's law and the continuity equation, partial differential equation governing one dimensional movement of water in the vertical direction is derived as

$$\frac{\partial\theta}{\partial t} = -\frac{\partial}{\partial z}\left(K(\psi)\frac{\partial\psi}{\partial z}\right) + \frac{\partial K(\psi)}{\partial z} \tag{1}$$

Where θ is volumetric water content, t is time, z is vertical coordinate pointing downward, $K(\psi)$ is hydraulic conductivity as a function of matric suction ψ. By adopting the diffusivity term and neglecting the gravity term, Eq.(1) can be rewritten as

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(D(\theta)\frac{\partial\theta}{\partial z}\right) \tag{2}$$

Where Exponential hydraulic conductivity model and exponential relation between θ and ψ (Teng et al., 2013) were widely adopted in analytical approach because they are able to linearize the partial differential equation with good results. The constitutive relations are expressed as follows:

$$K(\psi) = K_s e^{-\alpha\psi} \quad ; \theta(\psi) = \theta_r + (\theta_s - \theta_r)e^{-\alpha\psi} \tag{3}$$

Where K_s is the saturated hydraulic conductivity, α is desaturation coefficient, θ_s and θ_r are saturated and residual water content, respectively. Here, normalized water content Θ is defined as

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{4}$$

Substituting the Eq. (5) into Eq. (6), Eq. (5) is achieved

$$\frac{\partial\Theta}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial\Theta}{\partial z}\right) \tag{5}$$

The constant rate stage is primarily considered. It is reasonable to assign $E(t)$ as a constant E in this stages since hydraulic transport always satisfies the evaporative demands at surface. Therefore, initial and boundary condition can be expressed as

$$\Theta(z,0) = \Theta_0(z)\,;\ \Theta(L,t) = 1\,;\ E = D(\theta_s - \theta_r)\left.\frac{\partial\Theta}{\partial z}\right|_{z=0} \tag{6}$$

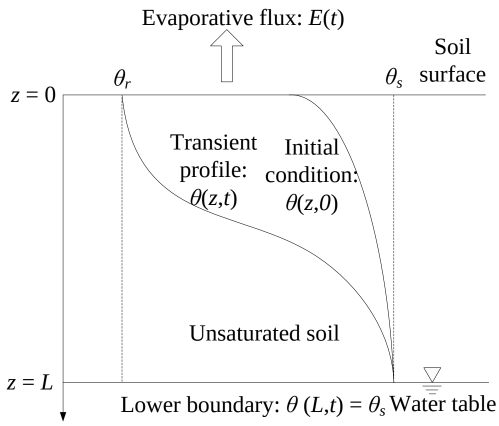

Figure 1 Schematic diagram of hypothetical water content distribution in unsaturated soil. θ_s is the water content at saturation, θ_r is the residual water content, $E(t)$ is time-dependent varying surface flux.

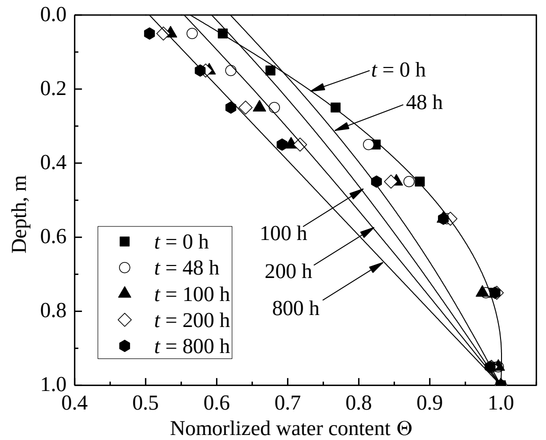

Figure 2 Simulated and measured water content profile for K-7 sand with water table of 1.0 m.

Where $\Theta_0(z)$ represents arbitrary initial condition. Finally, we can derive the solution of Eqs. (5) and (6) for modeling water content redistribution as expressed in Eq. (7):

$$\Theta(z,t) = 1 - \frac{2E\sqrt{Dt}}{(\theta_s-\theta_r)D}\sum_{m=0}^{\infty}(-1)^m\left\{\mathrm{ierfc}\frac{2mL+z}{2\sqrt{Dt}} - \mathrm{ierfc}\frac{2(m+1)L-z}{2\sqrt{Dt}}\right\} + \sum_{m=0}^{\infty}\left\{\frac{4(-1)^{m+1}}{(2m+1)\pi} + \frac{2}{L}\int_0^L \Theta_0(z)\cos\frac{(2m+1)\pi z}{2L}dz\right\}\exp\left\{-\frac{(2m+1)^2\pi^2 Dt}{4L^2}\right\}\cos\frac{(2m+1)\pi z}{2L} \tag{7}$$

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We assume t_c is the critical time between constant rate stage and falling rate stage. Whenever time is greater than critical time ($t > t_c$), the normalized water content Θ of soil surface persists at zero, which performs the upper boundary condition. Let $\bar{t} = t - t_c$, The initial and boundary conditions in this stage can be written as

$$\Theta(z,0) = f(z)\,;\quad \Theta(L,\bar{t}) = 1;\quad \Theta(0,\bar{t}) = 0 \tag{8}$$

Where $f(z)$ is water content distribution at $\bar{t} = 0$, it can be determined by substituting $t = t_c$ into Eq. (7). Referring to the derivation of Carslaw and Jaeger (1959), the solution of Eqs. (2) to (8) can be written as

$$\Theta(z,\bar{t}) = \frac{z}{L} + \frac{2}{\pi}\sum_{m=1}^{\infty}\frac{\cos m\pi}{m}\sin\frac{m\pi z}{L}e^{-Dm^2\pi^2\bar{t}/L^2} + \frac{2}{L}\sum_{m=1}^{\infty}\sin\frac{m\pi z}{L}e^{-Dm^2\pi^2\bar{t}/L^2}\int_0^L f(z')\sin\frac{m\pi z'}{L}dz' \tag{9}$$

3. Experimental verification

To verify the proposed analytical approach, two distinct soil evaporation tests would get introduced. The first one was carried out by authors, while the other one is referred from literature, which was performed by Song et al. (2013). Both two tests were performed by controlling climate conditions to investigate soil water evaporation. In authors' test, the soil evaporation column was instrumented in two cases: 150 cm in height and 200 cm in height. Different water moisture probes were installed along the center axis of column to measure the instantaneous volumetric water content. K-7 sand was adopted as the soil sample. Dry sand was packed into the column as uniformly as possible to achieve a dry density of 1.45 g/cm^3. Before the evaporation test, the soil column was wetted to saturation by supplying distilled water to bottom of soil column to avoid trapping air. After saturating soil sample, soil column was allowed to drain water until achieving equilibrium. The second evaporation test was carried out in Laboratoire Navier, Université Paris-Est, France on basis of large scale environmental chamber as described by Song et al. (2013). An 11.5 day evaporation test was conducted on Fontainebleau sand.

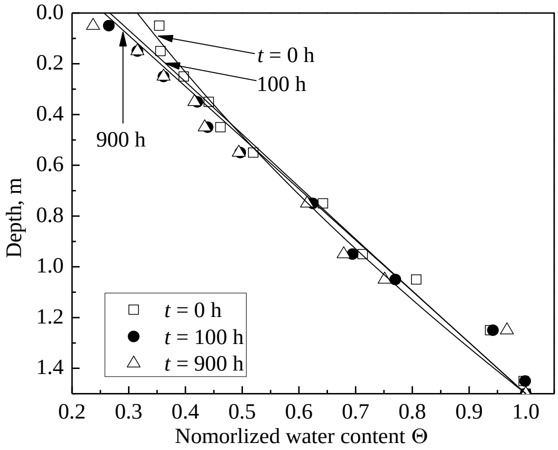

Figure 3 Simulated and measured water content profile for K-7 sand with water table of 1.5 m.

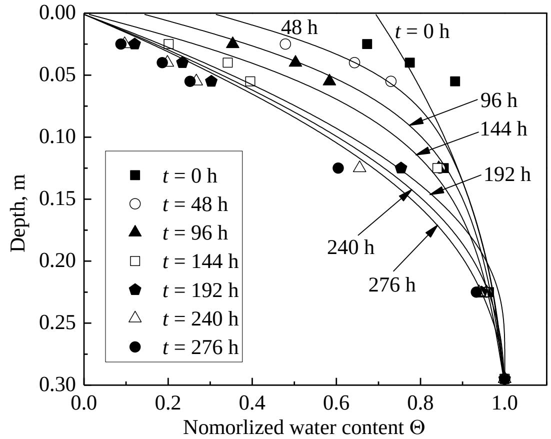

Figure 4 Simulated and measured water content profile for Fontainebleau sand.

Fig. 2 reports the simulated and measured profiles of water content at five times during evaporation test with water table of 1.0 m. it is observed that the water content variation mainly occurred in the top 0.45 m. Θ had a large decrease of about 10% for the first 200 h. After that, water content remained almost constant with achieving a steady state. this figure elucidates that the simulated water content profiles cannot fully coincides well with the measured one, it provides a way to approximate the extent of water content change. Fig. 3 presents the simulated and measured water content profiles with water table of 1.5 m. it is observed that water content didn't change much for all the depths, the measured maximum reduction occurred at the top 5 cm, which decreased from 35% to 24%. Comparing the computed profiles with the measured data, a reasonable agreement is obtained. The simulated and measured water content profiles of Fontainebleau sand are depicted in Fig. 4. Comparing the measured profiles with the computed ones at constant rate stage that are t = 48 h, 96 h, and 144 h, high agreement can be concluded. For water content profiles at time over 148.80 h, It is found that the proposed model overestimates the water content of soil at top 10 cm, the gap between estimation and measurement is about 8 % in average. Nevertheless, the analytical model is capable of providing an accurate prediction of water content redistribution at constant rate stage, while an approximate estimation at falling rate stage.

4. CONCLUSIONS

In this paper, we presented an isothermal model to simulate the water content redistribution during evaporation process. It is an analytical solution of linearized Richards equation based on exponential water retention and hydraulic conductivity relationship. In this model, transformation of constant rate stage into falling rate stage of evaporation is considered according to the surface soil water content. Also, the initial water content can be arbitrary to approximate the actual condition. The result of laboratory evaporation experiment together with literature data was utilized to validate the proposed model, results show that the proposed model can reasonably predict any temporal water content profile during evaporation process, but some discrepancies still exist.

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