ONE-DIMENSIONAL MODELING OF AIR POCKET ENTRAPMENT IN RECTANGULAR DUCTS

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1. INTRODUCTION

Sudden opening and closing the valves or starting and stopping the pumps can cause hydraulic transient from free surface-pressurized flow or pressurized-free surface flow (mixed flow) in ducts. This phenomenon can cause pumps or valves failures and duct breaches. This paper deals with a fundamental numerical model to simulate the confined air cavity that occurs under some flow conditions in ducts. Previous studies by Hosoda et al. (1994) and Hosoda et al. (2014) show this model can be applicable to reproduce air cavity in rectangular ducts. In this study, the numerical model has considered the pressure change of confined air to reproduce better entrapped cavity water surface profile in ducts. To evaluate the model, the simulation results compared with the experimental data of Baines (1991). Baines (1991) has conducted a series of experiments using a duct 10 cm square mounted at slopes from horizontal to 8˚. The length of the pipe was 4 m and he used weirs with 5cm and 7.5cm high to produce finite volume cavity.

2. NUMERICAL MODEL

The numerical model is composed of the continuity and momentum equations of open channel free surface, pressurized, and interface flows. Free surface flows, along with hypotheses on homogeneous and incompressible fluid, can be expressed by the partial differential continuity and momentum equations as follows:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0 \tag{1}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial uQ}{\partial x}+gA\cos\theta\frac{\partial h}{\partial x}=gA(\sin\theta-\frac{\tau_b}{\rho gR})-\frac{\partial \overline{u'^2}A}{\partial x} \tag{2}$$

where A = flow area; Q = flow rate; u = depth-average velocity; h = flow depth; τ_b = wall shear stress vector; R = hydraulic radius; $\overline{u'^2}$ = turbulence intensity; θ = duct bottom angle; ρ = water density; g = gravity acceleration; x = spatial step; and t = time step.

The wall shear stress is evaluated by Manning's formula (Eq. 3) and the eddy diffusity term is evaluated by Eq. (4). In this study, Manning roughness coefficient n = 0.01.

$$\tau_{bx}=\frac{\rho g n^2 u|u|}{h^{1/3}} \tag{3}$$

$$\frac{-\partial \overline{u'^2}A}{\partial x}=\frac{\partial}{\partial x}(AD_h\frac{\partial u}{\partial x});\qquad D_h=\alpha h|u|,\quad \alpha=0.05 \tag{4}$$

For pressurized flow, Eqs. (1) and (2) are transformed to obtain continuity and momentum equations as follow:

$$Q=Au \tag{5}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial uQ}{\partial x}+gA\frac{\partial}{\partial x}(\frac{P_D}{\rho g}+D\cos\theta)=gA(\sin\theta-\frac{\tau_b}{\rho gR})-\frac{\partial \overline{u'^2}A}{\partial x} \tag{6}$$

where D = pipe diameter; and P_D = pressure at top of pipe.

The momentum equation at an interface is derived by integration of Eq. (2) and (6) over the control volume (Hosoda et al. 1994).

$$\begin{aligned}&\frac{\partial Q}{\partial t}\Delta x+(uQ)_{x_{i+1/2}}-(uQ)_{x_{i-1/2}}+gA_{x_i}\left\{h_{i+1/2}-(\frac{P_D}{\rho g}+D)_{i-1/2}\right\}\frac{\partial}{\partial x}=\\&gA_{x_i}\Delta \mathrm{x}(\sin\theta-\frac{\tau_b}{\rho gR})_{x_i}-\frac{\partial \overline{u'^2}A}{\partial x}+(-\overline{u'^2}A)_{x_{i+1/2}}-(-\overline{u'^2}A)_{x_{i-1/2}}\end{aligned} \tag{7}$$

The effects of vertical acceleration of flow which causes the non-hydrostatic distribution of fluid pressure should be included in the momentum equation of open channel flow. Thus, the Boussinesq model follows (Hosoda et al. 2014)

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$$\frac{\partial Q}{\partial t}+\frac{\partial uQ}{\partial x}+gA\cos\theta\frac{\partial h}{\partial x}+\frac{\partial}{\partial x}\left(\frac{1}{3}h^2u^2\frac{\partial^2 A}{\partial x^2}+\frac{2}{3}h^2u\frac{\partial^2 A}{\partial t\partial x}+\frac{1}{3}h^2\frac{\partial^2 A}{\partial t^2}\right)$$
$$+\frac{\partial}{\partial x}\left(\frac{1}{3}h^2\frac{\partial u}{\partial t}\frac{\partial A}{\partial x}-\frac{1}{3}hu^2\frac{\partial h}{\partial x}\frac{\partial A}{\partial x}-\frac{1}{3}hu\frac{\partial h}{\partial x}\frac{\partial A}{\partial t}\right)=gA\left(\sin\theta-\frac{\tau_{bx}}{\rho gR}\right)+\frac{\partial(-\overline{u'^2}A)}{\partial x} \tag{8}$$

To take account of the air pressure change inside the confined cavity, Boyl's law was considered by

$$P_aV_a{}^{\gamma}=P_0V_0{}^{\gamma};\ \ P_a=P_0+\Delta P_a;\ \ \Delta P_a=P_0\left(\left(\frac{V_0}{V_a}\right)^{\gamma}-1\right) \tag{9}$$

where P_0 = atmospheric pressure; P_a = air pressure in confined cavity; V_0 = volume before air is confined; V_a = volume after air is confined; ΔP_a = differences between confined air pressure and atmospheric pressure; and γ = specific weight (constant).
The following term was added to the open channel free surface momentum equation to consider the air pressure change

$$-\frac{\partial}{\partial x}\left(\frac{\Delta P_a}{\rho}A\right)+\frac{\Delta P_a}{\rho}B_s\frac{\partial h}{\partial x} \tag{10}$$

where B_s = surface width inside the cavity.

3. RESULTS AND CONCLUSIONS

We tried to simulate the confined cavity observed in the experiments of Baines (1991) using a finite volume method. HSMAC method with pressure iteration procedure is applied to the pressurized flow region. The simulations were done by using Δx = 0.02 and Δt = 0.001. Fig 1 shows two cavities on 2˚ slope, w = 5cm. Fig. 2 and 3 show the simulation results for experiments on 2˚ slope and with w = 5cm and 7.5cm.

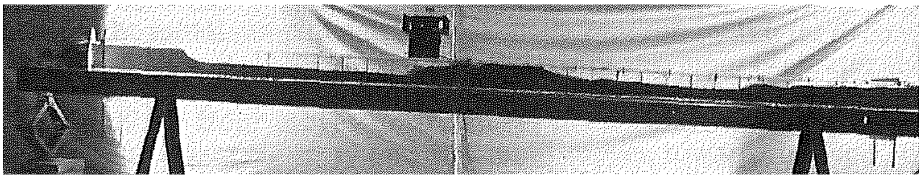

Fig. 1 Photo of Two Cavities on 2˚ Slope, w = 5cm (Baines 1991)

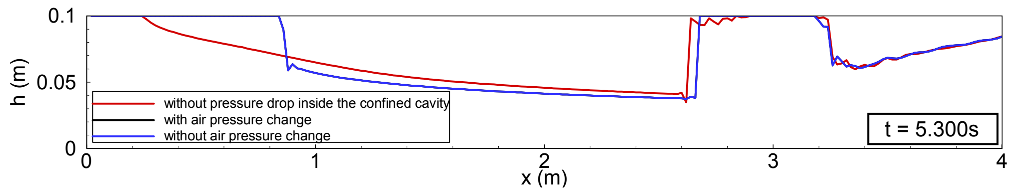

Fig. 2 Simulation Results for Cavities on 2˚ Slope, w = 5cm (Baines 1991)

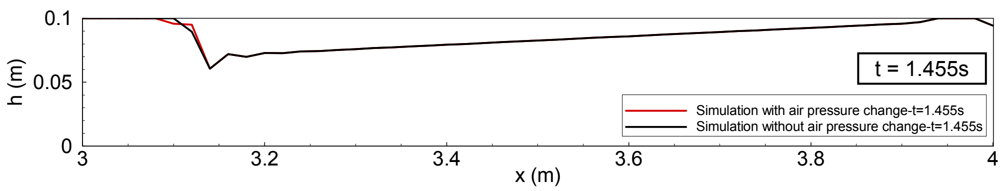

Fig. 3 Simulation Results for Cavities on 2˚ Slope, w = 7.5cm (Baines 1991)

It is pointed out through the comparisons between the simulated results and experiments that it is necessary to consider the pressure change inside the confined cavity to simulate the finite cavity volume in a duct. In addition, the results show that to reproduce the confined cavity profile, it is important to neglect the pressure drop inside the cavity.

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