Modal Identification of a Structure using Cross Correlation of Microtremors

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1. Introduction

Evaluating the structural perfomance of the existing structures is important to confirm the structural soundness after natural disaster or validity of the seismic retrofit. Modal identification techniques based on vibration records are usually applied to evaluate dynamic characteristics of the target structure. In this technique, we will determine natural frequency, modal vector and modal damping ratio, called modal characteristics as parameters. For architectures and civil engineering structures, modal identification method using microtremor records is desirable, since these records can be observed readily and frequently.

Although the modal identification is generally implemented based on frequency response function which can be obtained from input-output relation, it is difficult to specify and record input vibrations into a structure from microtremor data. Some alternative approaches have been proposed, which is premised on using only output records without the use of the input records [1, 2]. Despite the difficulty of input specification, many of methods, which use only output motions, assume input vibrations as whitenoise. Therefore, accuracy of parameters identified by these methods depends on whether the assumption of input motions are satisfited.

We will propose a new technique for the modal identification, which does not require any assumption to the input vibrations. Cross correlation analysis is applied to three records in order to calculate spectral ratio between records at two target sites. A technique for modal identification is presented on the basis of the cross correlation analysis. The method is applied to numerical simulation data and observed records on an actual structure, and its applicability is confirmed.

2. Theoretical Background of Proposed Method

2.1 Cross Correlation Analysis

In the proposed method, influence of input characteristics can be neglected by taking a spectral ratio of simultaneous observation records. Noise elimination should be considered to calculate accurate spectral ratio with respect to common inputs, since observed records include observation noise. Three records are used to calculate a cross spectral ratio, in which two of them are obtained at target sites and the another one is obtained at an arbitrary site in the structure. Let the Fourier spectrum at target sites p and q and another site l as $Y_p(\omega)$, $Y_q(\omega)$ and $Y_l(\omega)$, the cross spectral ratio is defined as

$$R_{pq}(\omega) = \frac{S_{pl}(\omega)}{S_{ql}(\omega)}, \tag{1}$$

where $S_{mn}(\omega) = \langle Y_m(\omega) Y_n^*(\omega) \rangle$ $(m,n = p,q,l)$ is cross spectrum between sites m and n, and $*$ and $\langle \cdots \rangle$ stand for complex conjugate and expectation in the meaning of emsemble, respectively.

2.2 Modal Identification Technique

Identification of the modal characteristics based on the result of cross spectral ratio $R_{pq}(\omega)$ defined in Eq.(1) is examined with the multi-degree-of-freedom (MDOF) systems, and the damping term is assumed as the proportional viscous damping.

If the modes can be separated in the vicinity of each natural frequency, the system can be approximated as 1 degree-of-freedom system. Under this approximation, the cross spectral ratio $R_{pq}(\omega)$ at r-th natural frequency ω_r can be represented as a modal amplitude ratio of the two sites:

$$R_{pq}(\omega_r) \approx \frac{u_{rp}}{u_{rq}}, \tag{2}$$

where u_{rp} and u_{rq} are the r-th modal vector at site p and q, respectively. Also, imaginary part of the cross spectral ratio $\Im[R_{pq}(\omega)]$ at r-th natural frequency ω_r has a sensitivity to the each modal damping ratio h_r.

From these results, following modal identification algorithm can be proposed: (1) Identify the natural frequency from Fourier amplitude spectrum, (2) Identify the modal vector from real part of the cross spectral ratio $\Re[R_{pq}(\omega)]$, (3) Identify the modal damping ratio by the forward calculation using imaginary part of the cross spectral ratio $\Im[R_{pq}(\omega)]$.

3. Applications

3.1 Numerical Simulation

The proposed method for modal identification is applied to numerically simulated microtremor data for 5DOF system to confirm its applicability. The non-white noise inputs which are uncorrelated each other and have a different frequency characteristic are imposed as a ground motion and external force to the 5th mass.

Keywords: Structural health monitoring (SHM), Modal characteristics, Microtremor, Cross correlation analysis
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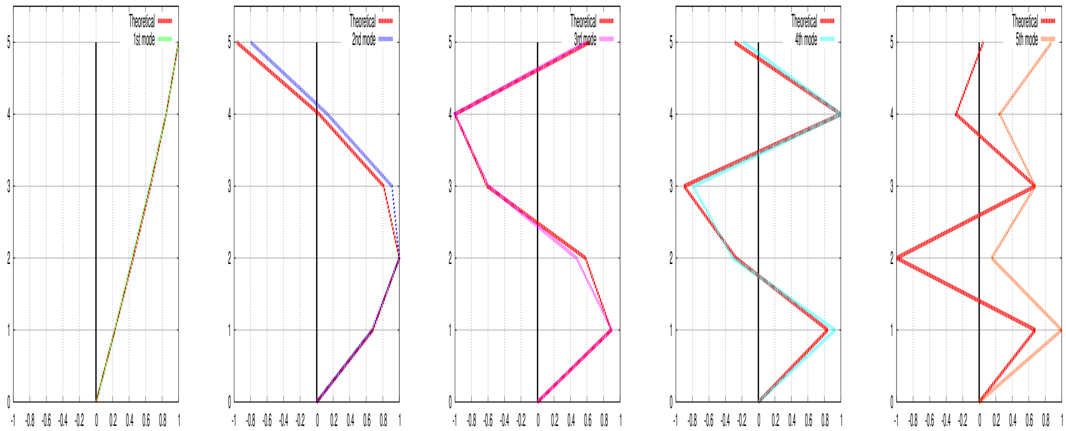
Table 1: Identified parameters (values in brackets are analytical values)

Mode number r	1	2	3	4	5
Natural frequency f_r(Hz)	2.834 (2.820)	7.665 (7.549)	11.858 (11.977)	15.493 (15.743)	19.860 (19.213)
Modal damping ratio h_r	0.005 (0.040)	0.070 (0.037)	0.056 (0.034)	0.053 (0.030)	– (0.027)

Table 2: Identified natural frequencies

Component	Short side		Long side	
Mode number r	1	2	1	2
Natural frequency f_r (Hz)				
1st Obs.	1.69	6.26	1.97	6.50
2nd Obs.	1.69	6.11	1.97	6.58



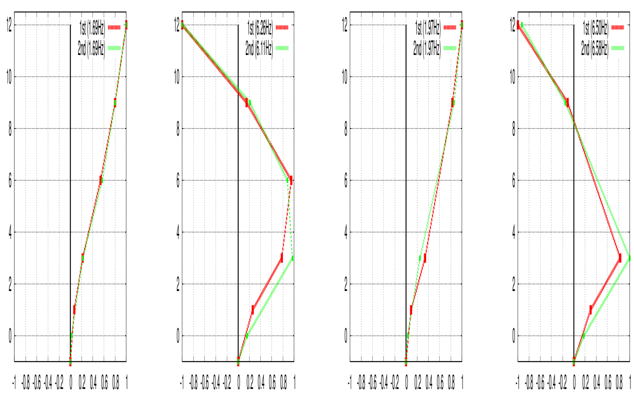
1st 2nd 3rd 4th 5th

Fig. 1: Identified modal vectors for a numerical example (red lines: analytical).

Identified natural frequencies f_r and modal damping ratios h_r are shown in Table 1 (values in brackets show analytical values). From the results of modal damping ratio h_r, it was difficult to obtain exact estimation from the proposed method. Also, identified modal vectors $\{u_r\}$ are shown in Fig.1 (red line shows analytical values). The modal vectors were accurately estimated by the proposed method except for the 5th mode vector. These results indicate that the proposed method can identify parameters without any information on input motions.

3.2 Data Observed at Actual Structure

The microtremor observation has been carried out at G3 building in Suzukake-dai campus (11-storey above ground and one below), and five sensors were installed with even intervals in the building. About one hour observation has held twice with a sampling rate of 200Hz.



1st 2nd 1st 2nd

Short side component Long side component

Fig. 2: Identified modal vectors for G3 building.

Here, natural frequencies f_r and modal vectors $\{u_r\}$ were identified by the proposed method. As shown in Table 2, identified natural frequencies using two obsevation data are almost identical. Fig.2 is the results of identified modal vectors. Red line and green line in each figure corresponds to the result based on 1st and 2nd observations, respectively. Since obtained modal vectors based on two data are almost identical, the identification has done properly.

4. Conclusion and Future Development

In this research, modal identification method based on cross spectral ratio is proposed as a method which uses only output motions. The proposed method can identify modal vectors without any information on input motions, and was applied to the observed records on the actual structure. However, since the difficulties are recognized in the identification of modal damping ratio, corresponding algorithms should be modified.

Reference

[1] Brincker, R., Zhang, L., and Andersen, P., “Modal identification of output-only systems using frequency domain decompositions,” *Smart Materials and Structures*, Vol.10, No.3, 2001, pp.441–445.

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