NUMERICAL MODELING OF WATER FLOW OVER EMBANKMENT USING LATTICE BOLTZMANN METHOD

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1. INTRODUCTION

The embankment failure events draw global attention in recent years because of the rapid increasing of storm rainfall, flood, tsunami etc. It is very complicated to analyze them because of the interaction among flow, erosion and the change of embankment profile. In this paper, lattice Boltzmann method for shallow water equations is used for estimating the water flow over embankment. And the empirical erosion equation is used for estimating the erosion starting point.

2. SHALLOW WATER EQUATIONS

Generally, the continuity and Navier-Stokes equations are used to describe incompressible fluid flows. But usually the water depth in rivers, channels and coastal areas is much smaller than the horizontal length, where the velocity can be approximated by uniform distribution along the depth. Meanwhile when the flow is subcritical, and the angle of bed slope is small, which is to say the sine of slope angle is approximately the same with the tangent value. Such kind of flows can be described by shallow water equations as

$$\frac{\partial h}{\partial t}+\frac{\partial (hu_j)}{\partial x_j}=0 \tag{1}$$

$$\frac{\partial (hu_i)}{\partial t}+\frac{\partial (hu_iu_j)}{\partial x_j}=-g\frac{\partial}{\partial x_j}(\frac{h^2}{2})+\nu\frac{\partial^2 (hu_i)}{\partial x_j\partial x_j}+F_i \tag{2}$$

where i and j are indices and the Einstein summation convention is used; h the water depth; t the time; u_i the depth-averaged velocity component in i direction; g the gravitational acceleration; ρ the water density; ν the kinematic viscosity; F_i is called “force term” here, and defined as:

$$F_i=-gh\frac{\partial z_b}{\partial x_i}-\frac{\tau_{bi}}{\rho} \tag{3}$$

where z_b the bed elevation above datum; and τ_{bi} the bed shear stress.

3. LATTICE BOLTZMANN METHOD

The lattice Boltzmann method is derived from lattice gas model and continuum Boltzmann equation from the view of mecroscopical partical movement, but it can reveal the physical fluid behavior. The distribution function is the only variable in the lattice Boltzmann method, and the easy implementation of boundary condition make the method to be a very promising approach in a variety of fields, especially in hydraulics and engineering. The most popular form is given by

$$f_\alpha(\mathbf{x}+\mathbf{e}_\alpha\Delta t,t+\Delta t)-f_\alpha(\mathbf{x},t)=-\frac{1}{\tau}(f_\alpha-f_\alpha^{eq})+\frac{\Delta t}{N_\alpha e^2}e_{\alpha i}F_i(\mathbf{x},t) \tag{4}$$

where f_α is the distribution function, $\mathbf{x}$ the lattice site, $\mathbf{e}_\alpha$ the lattice direction, τ the relaxation time, f_α^{eq} the local equilibrium distribution function, N_α the constant decided by lattice pattern, e the lattice velocity, F the force term. According to the theory of lattice Boltzmann method and shallow water equation, the water depth and the velocity can be defined in terms of the distribution function as

$$h=\sum_\alpha f_\alpha, \quad u_i=\frac{1}{h}\sum_\alpha e_{\alpha i}f_\alpha \tag{5}$$

4. EMBANKMENT EROSION

Visser (1998) has investigated the embankment failure processes and divided them into five stages, Fujisawa (2009) described the processes theoretically by Froude-critical point migration. Accordingly, the erosion rates of cohesive soil are empirically defined as the following form:

$$E=\alpha(\frac{\tau_b}{\tau_{th}}-1)^\gamma \tag{6}$$

where E is the soil erosion rate, α and γ are the material properties have the dimension of velocity, τ_b is the boundary shear

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stress, and τ_{th} is the threshold boundary shear stress.

5. NUMERICAL RESULTS

To verify the lattice Boltzmann method for shallow water equation, the benchmark problem of 2D channel flow is used. With 100×50 square lattices, $\Delta x = \Delta y = 0.02m$, initial water depth is $2.5m$ and velocity is $12.5m/s$. Therefore the lattice velocity $u = 0.25$, depth $h = 0.05$, relaxation time $\tau = 1.5$ and the Manning coefficient is chosen as earth dam, 0.025. And the bounce back scheme for boundary condition was used, which means the incoming particles toward the boundary will bounce back into fluid, therefore the velocity at the boundary is neglected. The results are shown in Fig.1, the flow is fully developed at about 5000 time steps, the velocity profile and the boundary shear stress profile both have the parabolic shape.

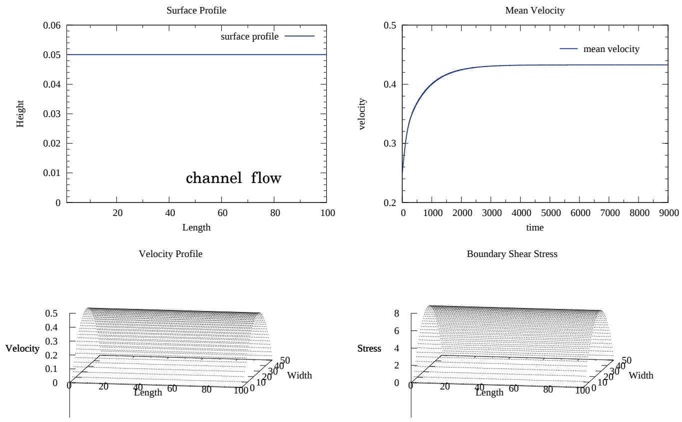

Figure 1: Channel Flow

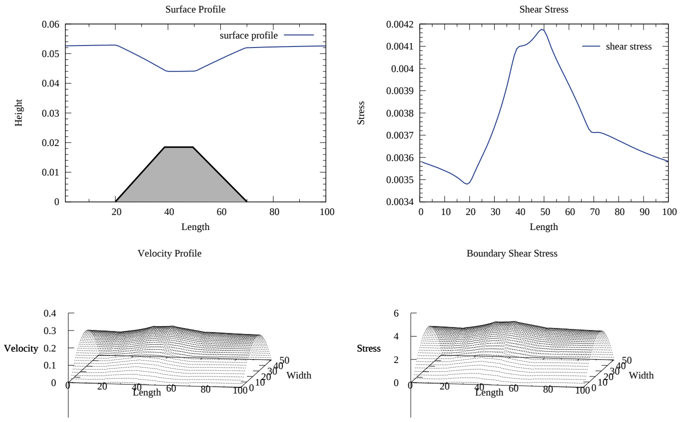

Figure 2: Flow Over Embankment

While for the specific embankment erosion, the shape of the embankment is shown in table1, with the same parameters and boundary conditions as in the channel flow case. The embankment elevation z_b was applied by the force term in Eq. 3, and the results are shown in Fig.2. Basically, when a steady flow pass over a embankment, there is surface drop above the embankment, and the figure just shows the same phenomenon. Take a close look at the boundary shear stress, when the flow pass the embankment, the largest boundary shear stress happens in the front of the crest as shown in Fig.2, according to the empirical erosion function Eq.6, the embankment erosion will start from the front crest.

x	0	20	40	50	70	100
z_b	0	0	0.02	0.02	0	0

Table 1: Embankment elevation z_b

6. CONCLUSIONS

It is concluded from the results that Lattice Boltzmann method is an effective approach in solving the shallow water equations. And shallow water equations can be regarded as a promising way in describing the embankment erosion problems. Based on the numerical results, the erosion will start from the front crest of the embankment.

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