A COMPUTATIONAL SIMULATION OF FRICTIONLESS CONTACT PROBLEM WITH LARGE DISPLACEMENT

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1. INTRODUCTION

Studies of contact problem have been widely executed by researchers with variable scopes, methods and definitions. A common problem occurs while handling contact phenomena is sliding through element boundary, discussed by Chen and Nakamura (1998), due to the discontinuity of the local coordinate between elements and a contact point. The common problem that occurs at an element boundary is a stable convergence result is hard to achieved, thus inspires authors to make a comparison of two different beam methods which are Euler-Bernoulli Beam Theory and Timoshenko Beam Theory for frictionless contact problem. Authors have been investigated geometrical non-linear analysis with extremely large displacements by using Tangent Stiffness Method, a robust non linear analysis method to execute analysis and produce results with high accuracy. In this study, a simple yet effective approach of a basic node to element contact phenomenon have been studied, which introduces a contact element consisted by three nodes. In this study, authors propose the modification of the beam elements with three nodes by considering the adaptation of shear deformation by TBT. The modification enables the contact point to slide through the element edge smoothly and some numerical examples are showed in this study.

2. TANGENT STIFFNESS METHOD

Here, let an element constituted by two edges with its element edge forces and the force vector for both edges is assumed as **S**. Let the external force vector as **U**, in a plane coordinate system with **J**, the equilibrium matrix, and the equilibrium condition could be expressed as the following equation.

$$\mathbf{U} = \mathbf{J}\mathbf{S} \tag{1}$$

With the differentiation of Eq. (1), the tangent stiffness equation could be expressed as;

$$\delta\mathbf{U} = \mathbf{J}\delta\mathbf{S} + \delta\mathbf{J}\mathbf{S} = \left(\mathbf{K_O} + \mathbf{K_G}\right)\delta\mathbf{d} \tag{2}$$

Here, the differentiation of Eq. (1) simultaneously extract $\delta\mathbf{S}$ and $\delta\mathbf{J}$ makes it possible to express a linear function of displacement vector, $\delta\mathbf{d}$ in the local coordinate system. Meanwhile, in Eq. (2), $\mathbf{K_O}$ represents the element stiffness matrix which also simulates the element behavior, correspondent to the element stiffness in the coordinate system while $\mathbf{K_G}$, the tangent geometrical stiffness represents the element displacement originated by the tangent geometrical stiffness.

3. COMPARISON OF EULER BEAM AND TIMOSHENKO BEAM FOR EXTREMELY LARGE INCREMENT OF LOADING

With the aforementioned method, TSM could solve any geometrically non-linear problem, even for extremely large deformation. Therefore, in this section, author will provide a comparison for bundled loading for both Euler beam and Timoshenko beam with a common plane frame structure.

$$\begin{bmatrix} N \\ M_i \\ M_j \end{bmatrix} = \begin{bmatrix} \frac{EA}{l_0} & 0 & 0 \\ 0 & \frac{4EI}{l_0} & \frac{2EI}{l_0} \\ 0 & \frac{2EI}{l_0} & \frac{4EI}{l_0} \end{bmatrix} \begin{bmatrix} \delta l \\ \theta_i \\ \theta_j \end{bmatrix}, \quad \begin{bmatrix} N \\ M_i \\ M_j \end{bmatrix} = \begin{bmatrix} \frac{EA}{l_0} & 0 & 0 \\ 0 & \frac{EI}{l_0(1+q)}(4+q) & \frac{EI}{l_0(1+q)}(2-q) \\ 0 & \frac{EI}{l_0(1+q)}(2-q) & \frac{EI}{l_0(1+q)}(4+q) \end{bmatrix} \begin{bmatrix} \delta l \\ \theta_i \\ \theta_j \end{bmatrix} \tag{3), (4}$$

$$q = \frac{12EI}{GAl_0} \tag{5}$$

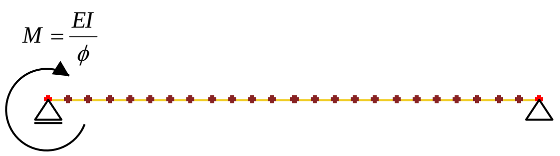

Figure 1 : Plane beam in pre-loading

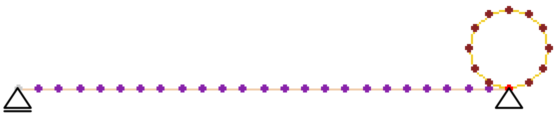

Figure 2 : Plane beam in post-loading

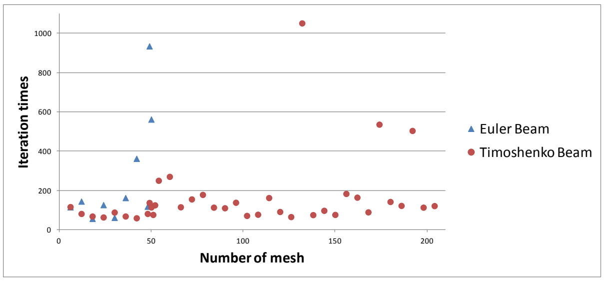

Figure 3 : The relation between number of mesh vs. iteration times

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Eq. (3) represents a common element force equation for Euler beam, while Eq. (4) shows Timoshenko beam theory with the consideration of shear deformation, shown in Eq. (5). Fig. (1) shows a simply supported beam with a roller support at one side and a pin support at the other. An extremely large moment load is applied in a single increment at the roller support until the beam deformed to a circular shape. The beam meshes is set from 6 to 200 meshes, and for this case, a stable convergence result for Euler beam is until 52 meshes, while for Timoshenko beam, stable convergence result has been achieved even until 200 meshes.

4. COMPARISON OF EULER AND TIMOSHENKO BEAM FOR FRICTIONLESS CONTACT PROBLEM

In this section, the element force equation for Euler and Timoshenko beam is introduced. Fig. 4 shows an equilibrium condition of an elastic and homogeneous simply supported beam which is subjected by axial force ***N***, edge moments $\boldsymbol{M_i}$ and $\boldsymbol{M_j}$, and contact force $\boldsymbol{Y_c}$. The element force equation for contact problem for Euler beam is shown in Eq. (6) and for Timoshenko beam is shown in Eq. (7). The Difference between these two theory is in Timoshenko beam, shear deformation (γ), is considered for the large deformational analysis.

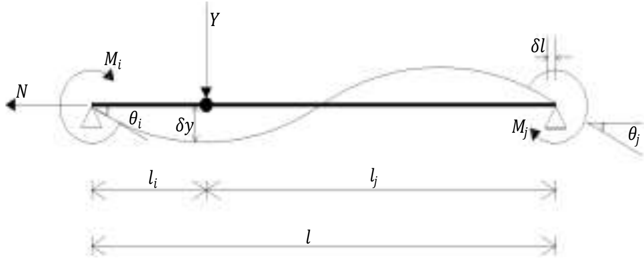

Figure 3: Contact problem in simply supported beam coordinate

$$\begin{bmatrix} N \\ M_i \\ M_j \\ Y_c \end{bmatrix} = \left\{ \frac{EI}{l_0} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \dfrac{4l_{ic}+3l_{jc}}{l_{ic}} & \dfrac{9l_0^2 - l_{ic}^2 l_{jc}^2}{l_{ic}^2 l_{jc}^2} & -\dfrac{3l_{jc} l_0^2}{l_{ic}^2 l_{jc}^2} \\ 0 & \dfrac{9l_0^2 - l_{ic}^2 l_{jc}^2}{l_{ic}^2 l_{jc}^2} & \dfrac{4l_{jc}+3l_{ic}}{l_{jc}} & \dfrac{3l_{ic} l_0^2}{l_{ic}^2 l_{jc}^2} \\ 0 & -\dfrac{3l_{jc} l_0^2}{l_{ic}^2 l_{jc}^2} & \dfrac{3l_{ic} l_0^2}{l_{ic}^2 l_{jc}^2} & \dfrac{3l_0^4}{l_{ic}^3 l_{jc}^3} \end{bmatrix} + \begin{bmatrix} \dfrac{EA}{l} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right\} \begin{bmatrix} \delta l \\ \theta_i \\ \theta_j \\ \delta y_c \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} N \\ M_i \\ M_j \\ Y_c \end{bmatrix} = \left\{ \frac{EI}{l_0 \Omega} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 4\Omega + 3l_{ic} l_{jc}^3 - 108\Psi^2 & 9\left(l_0^2 - l_{jc}\Psi\right) - \Omega & -3l_0\left(l_{jc} l_0 + 6\Psi\right) \\ 0 & 9\left(l_0^2 - l_{jc}\Psi\right) - \Omega & 4\Omega + 3l_{ic}^3 l_{jc} - 108\Psi^2 & 3l_0\left(l_{ic} l_0 + 6\Psi\right) \\ 0 & -3l_0\left(l_{jc} l_0 + 6\Psi\right) & 3l_0\left(l_{ic} l_0 + 6\Psi\right) & \dfrac{3l_0\left(l_0^3 + 12 l_0 \Psi\right)}{l_{ic} l_{jc}} \end{bmatrix} + \begin{bmatrix} \dfrac{EA}{l} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right\} \begin{bmatrix} \delta l \\ \theta_i \\ \theta_j \\ \delta y_c \end{bmatrix} \tag{7}$$

$$\Psi = \frac{EI}{GA} \qquad \Omega = \left(l_{ic} l_{jc}\right)^2 + 3\Psi\left(l_0^2 + l_{ic} l_{jc}\right) + 36\Psi^2$$

5. NUMERICAL EXAMPLE

In this analysis, a cantilever beam configuration as shown in Fig. 6 is applied for contact analysis. The beam is consisted by 18 segments and 19 nodes. A compulsory displacement in upwards direction is applied on a contact node, which is marked with the red node. In this analysis, we will investigate the territory which leads to divergence of the unbalanced force. Fig. 5 relation between percentage of l_i/l in a single contact element and the displacement of contact node in post-contact condition. In this analysis, the contact node position is set in six different positions, which are 4.05 *m*, 4.1 *m*, 4.2 *m*, 4.3 *m*, 4.35 *m* and 4.4 *m* in horizontal direction.
From the analysis result, it is significantly clear that by the consideration of shear deformation in Timoshenko beam, a stable yet converged solution have been successfully achieved at the edge of the segment which ranges from 99.499% to 99.933%. For Euler beam, the percentage ranges from 87.408 % to 92.251% and for cantilever coordinate, it ranges from 96.135% to 97.836%

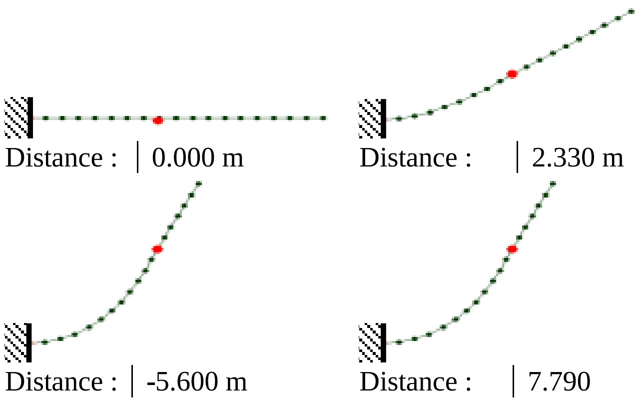

Figure 4 : Contact phenomena of a cantilever beam

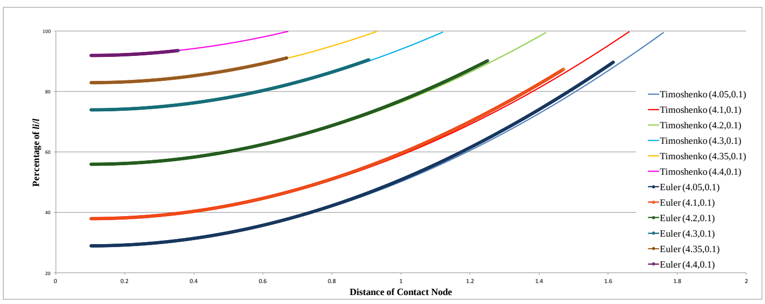

Figure 5 : Relation between percentage of l_i/l

5. CONCLUSION

For these two theories, unbalanced force will either diverge or no convergence result could be achieved beyond aforementioned ranges. On the other hand, for Timoshenko beam, unbalanced force is steadily converged around the tip of the segment, and the contact node is able to slide through to the next segment.