Large Scale Nonlinear Computation in Application to Seismic Analysis of RC Pier

University of Tokyo Student Member Fangtao Sun*
National Research Institute for Earth Science and Disaster Prevention Regular Member Takuzo Yamashita
University of Tokyo Regular Member Seizo Tanaka, Muneo Hori,
Keio University Regular Member Kenji Oguni

1. Introduction

In this paper, a 3D large-scale FEM nonlinear analysis in application to a RC bridge pier with fine resolution is conducted. A parallel program ADVENTURECluster[4] based on ADVENTURE project is employed in this study to reduce the computational time and to distribute the memory usage. For a more accurate material model, Maekawa's elasto-plastic fracture model of concrete is employed. Its nonlinear concrete constitutive relations are reformulated so that a fast solver for large-scale nonlinear FEM is applicable. Meanwhile, in order to simulate the crack propagation in concrete, the Particle Discretization Scheme Finite Element Method (PDS-FEM) is also adopted into this study.

2. Reformulation of nonlinear constitutive relations of concrete

In the elasto-plastic fracture model of concrete[1], the essence is a set of the elastic strain-stress relation and the elastic strain–plastic strain increment relation, based on the decomposition of strain into elastic and plastic parts. When the elastic strain and stress are denoted by $\boldsymbol{\epsilon}^E$ and $\boldsymbol{\sigma}$, the set of equations are expressed as follows:

$$\boldsymbol{\sigma} = \mathbf{c} : \boldsymbol{\epsilon}^E, \quad d\boldsymbol{\epsilon}^P = \boldsymbol{\ell} : d\boldsymbol{\epsilon}^E \tag{1}$$

Here, $\mathbf{c}$ is isotropic elasticity tensor, which is a forth-order tensor-valued nonlinear function of $\boldsymbol{\epsilon}^E$, and $\boldsymbol{\ell}$ is another forth-order function of $\boldsymbol{\epsilon}^E$.

From the above two equations, the $d\boldsymbol{\sigma}$-$d\boldsymbol{\epsilon}$ relation is easily derived as follows:

$$d\boldsymbol{\sigma} = \mathbf{c}^{EP} : d\boldsymbol{\epsilon} \tag{2}$$

$$\mathbf{c}^{EP} = \left(\mathbf{c} + \nabla\mathbf{c} : \boldsymbol{\epsilon}^E\right) : (\mathbf{I} + \boldsymbol{\ell})^{-1} \tag{3}$$

Where, $\mathbf{c}^{EP}$ is the forth-order elasto-plasticity tensor, $\nabla\mathbf{c} : \boldsymbol{\epsilon}^E$ is a forth-order tensor of its component being $(\partial c_{ijpq}/\partial \epsilon^E_{kl})\epsilon^E_{pq}$.

Numerical computation of $\mathbf{c}^{EP}$ requires large computational load. Computation of inverse tensor $(\mathbf{I} + \boldsymbol{\ell})^{-1}$ is needed. Furthermore, as damage and plastic deformation proceeds, $\mathbf{c}^{EP}$ loses its positive definiteness. The global stiffness matrix which is constructed from it also loses positive definiteness.

In order to reduce the computational cost, $\mathbf{c}^{EP}$ is simplified using the associated flow rule with a yield function $J_2^P = H(J_2^E)$.

$$\mathbf{c}^{EP} = \left(\mathbf{c} + \nabla\mathbf{c} : \boldsymbol{\epsilon}^E\right) : (\mathbf{I} - \mathbf{L}) \tag{4}$$

Here, $\mathbf{L} = \mathbf{d} \otimes \frac{H'}{2(1+H')}\frac{\mathbf{e}^E}{J_2^E}$. The simplified $\mathbf{c}^{EP}$ does not involve the inverse tensor.

In order to make a fast solver such as CG become applicable for large scale nonlinear FEM program. The constitutive relations are reformulated as[2],

$$d\boldsymbol{\sigma} = \mathbf{c} : \boldsymbol{\epsilon} + d\boldsymbol{\sigma}^* \tag{5}$$

Here, $d\boldsymbol{\sigma}^* = -\mathbf{c} : d\boldsymbol{\epsilon}^P + \left(\nabla\mathbf{c} : \boldsymbol{\epsilon}^E\right) : d\boldsymbol{\epsilon}^E$. Since $\mathbf{c}$ is always symmetric and positive definite, it is guaranteed that the global stiffness matrix is symmetric and positive definite. The second term on the right side is regarded as the explicit value from previous step when used in iteration. Therefore, a fast and efficient solver can be applied.

3. Formulation of failure analysis

Particle Discretization Scheme (PDS)[3] is employed in order to simulate crack propagation in concrete structures. PDS uses two sets of non-overlapping characteristic functions of Voronoi blocks and Delaunay tessellations to discretize a function f say displacement and its derivative ∇f. The filed variables can be expressed as,

$$f(\mathbf{x}) = \sum_{\alpha} f^{\alpha}\phi^{\alpha}(\mathbf{x}), \quad \nabla\mathbf{f}(\mathbf{x}) = \sum_{\beta} \nabla\mathbf{f}^{\beta}\psi^{\beta}(\mathbf{x}) \tag{6}$$

Here, $\{\phi^{\alpha}\}$ and $\{\psi^{\beta}\}$ are non-overlapping characteristic functions of Voronoi blocks and Delaunay tessellations, respectively. Since displacement filed is discretized with discontinuous shape functions, PDS provides a simple and numerically efficient failure treatment.

* Email: sun@eri.u-tokyo.ac.jp

4. Application

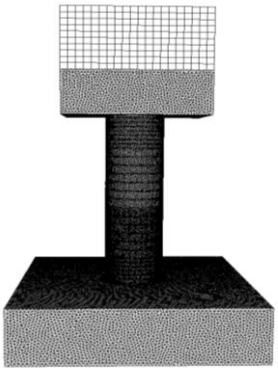

Fig. 1 FE model of RC pier

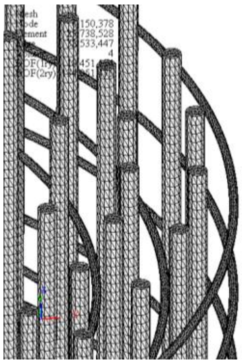

Fig. 2 FE model of rebars

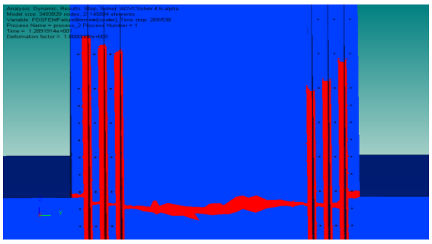

Fig. 3 Crack propagation at 5.39s

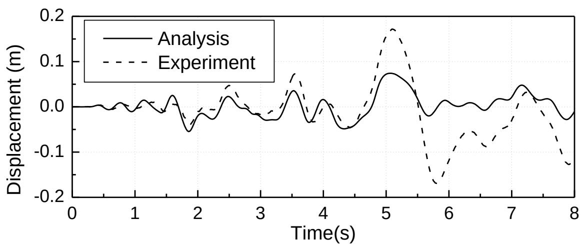

Fig. 4 Displacement time-history of RC pier in longitutal direction

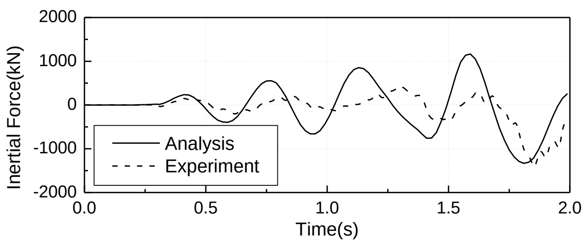

Fig. 5 Inertial force time-history of RC pier in longitutal direction

A 3D detailed finite element analysis model of a RC bridge pier cylinder with height of 6.0 meter and diameter of 1.8 meter was constructed (see Fig.1). Fig.2 shows main reinforcing bars and stirrups located inside the cylinder pier. The total number of nodes is 3,493,929; the number of tetrahedral elements is 21,145,594 and the number of degree of freedom is 10,481,787. Reformulated nonlinear concrete constitutive relations are used for concrete material and elasto-plastic hardening model is adopted for rebar. The earthquake wave at JR Takatori station of Hanshin-Awaji earthquake was selected as the input wave, which was considered as the forced displacement at the bottom of the model.

Crack occurs during the simulation process. Crack state at 5.39s is shown in Fig.3. Fig. 4 illustrates the displacement time-history of top surface of RC pier in longitutal direction. It is observed that there are differnces between numerical results and experimental results even in the linear response stage.

In the experiment, instruments are fixed around the top of RC pier to prevent it falling down. In oder to study the influence of the friction between the instruments and the RC pier, inertial force time-history was checked by setting the displacement at the top surface to be the same as the measured values in the experiment. Fig. 5 shows the inertial force time-history which also show difference between numerical results and experimental results. The causes of differences will be discussed in the future work.

5. Conclusion

Large-scale parallel program with nonlinear concrete constitutive relations and PDS is developed. It is a potentially important method to predict the dynamic response of structures under earthquakes and could play a critical role in preventing significant loss during seismic disasters. The program will be further improved and validated in the future.

References

1) Maekawa, K., Pimanmas, A., Okamura, H.. Nonlinear mechanics of reinforced concrete. *Taylor & Francis Group*. 2003

2) Yamashita, T., Hori, M., Oguni, K., et al. Reformulation of non-linear constitutive relations of concrete for large-scale finite element method analysis. *Journal of Japan Society of Civil Engineers*, Ser. A2 (Applied Mechanics (AM)). Vol. 67, No. 1 pp. 145-154, 2011

3) Hori, M., Oguni, K., Sakaguchi, H.. Proposal of FEM implemented with particle discretization for analysis of failure phenomna, *Journal of the Mechanics and Physics of Solids*, Vol. 53, pp. 681-703, 2005

4) ADVENTURECluster. http://www.alde.co.jp