MDOF MODELING OF COUPLED TRANSMISSION LINE-TOWER SYSTEM

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1. INTRODUCTION

This paper proposes multi-degree-of-freedom (MDOF) model of coupled transmission line-tower system. The entire transmission structure systems composed of numerous lines and towers are impossible to analyze. Therefore one coupled section of entire system with one-span transmission lines and supporting towers may be selected as representative of the transmission system. In literature the coupled structures are treated by finite element method and experimental studies under dynamic loadings. Both are, however, tedious and time-consuming approaches but accurate. In order to come over these difficulties, simpler model for coupled system including one tower and connecting lines is examined in this study. The lumped masses are formed for lines and tower to achieve coupled mass and stiffness matrices which are essential for eigenvalue and dynamic analyses. Fig. 1 illustrates the coupled system investigated in this paper.

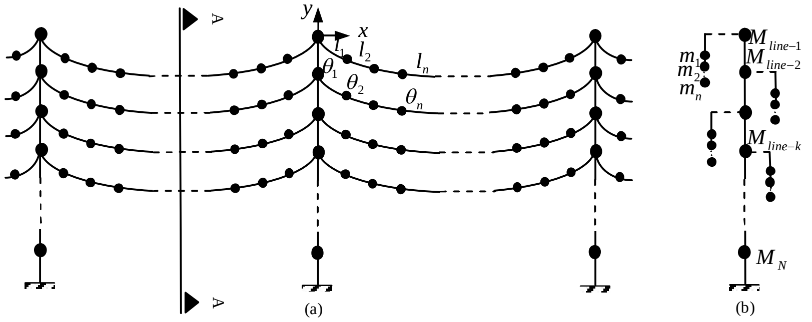

Fig. 1 Coupled system (a) in-plane direction (b) A-A cross-section (out-of-plane direction)

2. MDOF SYSTEM OF TRANSMISSION LINES

2.1 In-plane Direction

The lumped masses of lines are modeled as catenaries. The mass matrix is derived from the partial differentiation of kinetic energy $\left(\partial T/\partial \theta_i\right)$ (Clough et al.(2003)) in which,

$$T=\sum_{i=1}^{n}\frac{1}{2}m_i\left(\dot{x}_i^2+\dot{y}_i^2\right)=T\left(\theta_1,\theta_2,\ldots,\theta_n,\dot{\theta}_1,\dot{\theta}_2,\ldots,\dot{\theta}_n\right) \tag{1}$$

where n the number of lumped masses, m lumped mass of cable, $\dot{x}$ horizontal velocity component $\left(\partial x/\partial \theta\right)$, $\dot{y}$ vertical velocity component $\left(\partial y/\partial \theta\right)$, θ is the angle of line segment from the vertical coordinate. Similarly potential energy of system is differentiated to obtain stiffness matrix $\left(\partial V/\partial \theta_i\right)$(Clough et al.(2003)) which is given by,

$$V=\sum_{i=1}^{n+1}m_i g y_i+\sum_{i=1}^{n+1}\frac{1}{2}\frac{EA}{l_i}\left(l_i-l_{ik}\right)^2=V\left(\theta_1,\theta_2,\ldots\theta_{n+1}\right) \tag{2}$$

where $n+1$ is the number of line segments, y_i distance to upper mass, l_i the length of line between two lumped masses, E Young's modulus, A sectional area, l_{ik} equals to $l_i\cdot\sin\theta_i$.

The above equation includes potential energy due to gravity force and internal stiffness which is quantified according to the geometric positions of line fragments. The transformation matrix for each line segments is given by,

$$k_{local}=\frac{EA}{l_i}\begin{bmatrix}\cos^2\theta_i & -\cos\theta_i\sin\theta_i & -\cos^2\theta_i & \cos\theta_i\sin\theta_i\\ -\cos\theta_i\sin\theta_i & \sin^2\theta_i & \cos\theta_i\sin\theta_i & -\sin^2\theta_i\\ -\cos^2\theta_i & \cos\theta_i\sin\theta_i & \cos^2\theta_i & -\cos\theta_i\sin\theta_i\\ \cos\theta_i\sin\theta_i & -\sin^2\theta_i & -\cos\theta_i\sin\theta_i & \sin^2\theta_i\end{bmatrix} \tag{3}$$

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K_{global} is developed as $(n \times n)$ matrix by vanishing the stiffness except the ones related to the displaced shape of line along horizontal direction.

2.2 Out-of-plane Direction

The mass matrix is diagonal matrix which elements are $diag[m_1, m_2, \ldots, m_n]$. Along corresponding direction the stiffness matrix for each line is $(n \times n)$ matrix as following,

$$K_{line} = g\begin{bmatrix} \frac{(m_1+m_2+\ldots m_n)}{l_1\cos\theta_1}+\frac{(m_2+\ldots m_n)}{l_2\cos\theta_2} & -\frac{(m_2+\ldots m_n)}{l_2\cos\theta_2} & 0 & \cdots & 0 \\ -\frac{(m_2+\ldots m_n)}{l_2\cos\theta_2} & \ddots & 0 & \vdots & \vdots \\ 0 & \vdots & \ddots & 0 & 0 \\ \vdots & \vdots & 0 & \frac{m_n}{l_n\cos\theta_n}+\frac{(m_{n-1}+m_n)}{l_{n-1}\cos\theta_{n-1}} & -\frac{m_n}{l_n\cos\theta_n} \\ 0 & \cdots & 0 & -\frac{m_n}{l_n\cos\theta_n} & \frac{m_n}{l_n\cos\theta_n} \end{bmatrix} \tag{4}$$

3. MDOF SYSTEM OF TRANSMISSION TOWER

The lattice tower includes of a lot of steel members in different geometric positions. The most practical approach for complex structures to obtain stiffness matrix is using the flexibility matrix (Murtagh et al. (2004)). The flexibility matrix elements are deformation values of i^{th} node under unit loading subjected to j^{th} node of structure. The inverse of flexibility matrix, $[F]^{-1}$, yields stiffness matrix, $[K]$, of the structure. The mass matrix is diagonal matrix which elements are lumped masses of tower.

4. COUPLED TRANSMISSION STRUCTURE

4.1 In-plane Direction

The coupling of mass and stiffness matrices are essential to perform eigenvalue and dynamic response analyses of the coupled system. The coupled mass matrix,

$$M_{in} = \begin{bmatrix} M_{line} & M_{couple} \\ M_{couple}^T & M_{tower} \end{bmatrix} \tag{5}$$

in which $M_{line} = diag[M_{line-1}, M_{line-2}, \ldots, M_{line-k}]$, $M_{tower} = diag[M_1, M_2, \ldots, M_N]$, M_{couple} is the coupled masses of lines with tower.

4.2 Out-of-plane Direction

The mass matrix is $[M] = [M_{line}\ M_{tower}]^T$ while the stiffness matrix is as following,

$$[K] = \begin{bmatrix} K_{line} & K_{coupling} \\ K_{coupling}^T & K_{tower} \end{bmatrix} \tag{6}$$

$K_{coupling}$ is the stiffness matrix of line segment connected to tower which elements are placed at the $(i, j)^{th}$ element where i and j are the lumped mass of line connected to tower and the line layer respectively (Chen et al. (2009)).

5. CONCLUSIONS

The fundamental mode of proposed coupled system is obtained as 1.485 Hz and 1.878 Hz for in-plane and out-of-plane vibrations respectively whereas it is found as 1.562 Hz and 1.945 Hz for in-plane and out-of-plane vibrations of finite element model created on TDAP software staying in 10% error range. The MDOF model of coupled transmission line-tower system is applicable for further dynamic analysis.

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