

**SIMPLIFIED CRITERION TO SPECIFY OVERALL INSTABILITY OF ELEVATED CONTINUOUS GIRDER BRIDGE SYSTEM**

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**1. INTRODUCTION:** In the current seismic design code <sup>1)</sup>, the safety of multi-span elevated continuous girder bridges is verified by checking the safety of respective piers under design horizontal seismic accelerations. It is specified in the code that the ultimate state of the entire bridge system is reached if only one pier reaches its ultimate state defined as limit-load instability state. However, in reality, instability of one pier does not always lead to the overall instability of an entire system. Therefore, in order to precisely evaluate the safety margin for a bridge system, it will be necessary to provide some criterion to identify the overall instability of the system. For this purpose, authors examined an applicability of a criterion based on Hill's elastic-plastic stability theory <sup>2)</sup> by assuming that the inertia forces can be treated as static forces. The validity of this criterion was verified for elevated continuous straight girder bridges<sup>3)</sup>. However, it was shown by numerical analysis that this criterion does not work in the case of elevated continuous curved girder bridges. This is because the dynamically deformed shapes of the curved girders under seismic accelerations include higher order elastic deformation modes that are not encountered under static load. In view of this fact, we, herein, propose a simplified criterion where the second-order works done by the external inertia forces is calculated by ignoring the elastic deformation of girders. Since the elastic deformation contributes to enhance the stability of the entire system, this simplified criterion underestimates the system stability. In order to apply this new criterion to practical design, its accuracy is first examined by making use of our previous data concerning an elevated straight girder bridge<sup>2)</sup> and, then, the validity and applicability of the new criterion are discussed in terms of a horizontally curved elevated girder bridge under bi-directional seismic accelerations.

**2. STABILITY CRITERION FOR ULTIMATE STATE:** The general form of the elastic-plastic stability criterion for a structural system under static loads is expressed in terms of incremental external force components  $(\Delta F_x^i, \Delta F_y^i, \Delta F_z^i)$  and incremental displacement components  $(\Delta u_x^i, \Delta u_y^i, \Delta u_z^i)$  at the point of application of the  $i$ -th force. With these quantities, the 2<sup>nd</sup> order external work for a structural system is defined as:

$$\Delta^2 W_A = \sum_{i=1}^n (\Delta F_x^i \Delta u_x^i + \Delta F_y^i \Delta u_y^i + \Delta F_z^i \Delta u_z^i) / 2 \quad (1)$$

The stability criterion<sup>2)</sup> generally classifies the equilibrium state of structural system as stable, critical and unstable according to whether  $\Delta^2 W_A > 0$ ,  $\Delta^2 W_A = 0$  or  $\Delta^2 W_A < 0$ .

Therefore, the ultimate state of structural system is identified by the first zero-crossing point of  $\Delta^2 W_A$  from positive to negative.

For a multi-span continuous elevated-girder bridge model with multiple degrees of freedom shown in Fig.1(a), this model can be divided into two parts as illustrated in Fig.1(b), that is, one superstructure and a set of piers, by applying internal incremental force and moment components as external components at the interface of the two parts. These internal incremental force and moment components denoted, respectively, as  $(\Delta F_{Px}^j, \Delta F_{Py}^j, \Delta F_{Pz}^j)$  and  $(\Delta M_{Px}^j, \Delta M_{Py}^j, \Delta M_{Pz}^j)$  are those acting at the top of the  $j$ -th pier. The increments of the corresponding translational and rotational displacement components at the top of the  $j$ -th pier are respectively expressed as  $(\Delta u_{Px}^j, \Delta u_{Py}^j, \Delta u_{Pz}^j)$  and  $(\Delta \theta_{Px}^j, \Delta \theta_{Py}^j, \Delta \theta_{Pz}^j)$ . As a result,  $\Delta^2 W_A$  of the entire bridge system can be divided into two parts, that is, the 2<sup>nd</sup> order work of the superstructure  $\Delta^2 W_S$  and the sum of the 2<sup>nd</sup> order works for all the piers denoted as  $\Delta^2 W_{\Sigma P}$ .

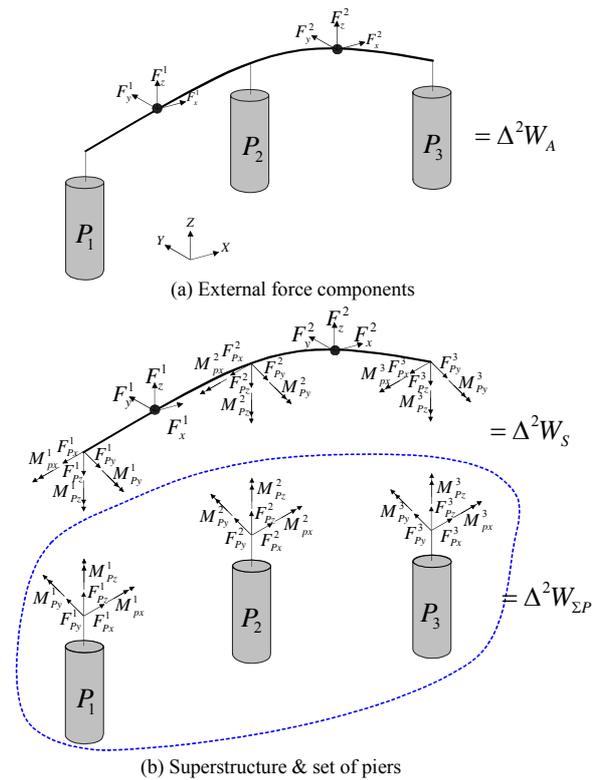
$$\Delta^2 W_A = \Delta^2 W_S + \Delta^2 W_{\Sigma P} \quad (2)$$

where

$$\Delta^2 W_S = \sum_{i=1}^n (\Delta F_x^i \Delta u_x^i + \Delta F_y^i \Delta u_y^i + \Delta F_z^i \Delta u_z^i) / 2 - \sum_{j=1}^m (\Delta F_{Px}^j \Delta u_{Px}^j + \Delta F_{Py}^j \Delta u_{Py}^j + \Delta F_{Pz}^j \Delta u_{Pz}^j + \Delta M_{Px}^j \Delta \theta_{Px}^j + \Delta M_{Py}^j \Delta \theta_{Py}^j + \Delta M_{Pz}^j \Delta \theta_{Pz}^j) / 2 \quad (3)$$

$$\Delta^2 W_{\Sigma P} = \sum_{j=1}^m (\Delta F_{Px}^j \Delta u_{Px}^j + \Delta F_{Py}^j \Delta u_{Py}^j + \Delta F_{Pz}^j \Delta u_{Pz}^j + \Delta M_{Px}^j \Delta \theta_{Px}^j + \Delta M_{Py}^j \Delta \theta_{Py}^j + \Delta M_{Pz}^j \Delta \theta_{Pz}^j) / 2 \quad (4)$$

In the current design, the plastification is not allowed in superstructure. Therefore, when displacement is relatively small,



**Fig. 1** Inertial forces that act over bridge system and piers

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$\Delta^2 W_s$  is alternatively expressed as  $\Delta^2 W_s \approx (\int_{V_s} E(\Delta \varepsilon)^2 dV_s) / 2 \geq 0$  where  $\int_V \cdot dV$  denotes integration over the volume of superstructure,  $\Delta \varepsilon =$  strain increment,  $E =$  Young's modulus. Considering that  $\Delta^2 W_s$  is always positive,  $\Delta^2 W_A \geq \Delta^2 W_{\Sigma P}$  holds. Based on this relation, a conservative criterion to identify the stability of elevated girder bridges can be expressed as  $\Delta^2 W_{\Sigma P} > 0$ . This criterion is also convenient because the 2<sup>nd</sup> order work done by the higher order elastic deformation modes of the superstructure is excluded. However, in order to apply this criterion to practical design, it is necessary to confirm that this new criterion is moderately conservative in order to avoid uneconomical design. In the next section, its accuracy and validity are examined by numerical examples.

**3. BRIDGE MODEL:** The accuracy of the new stability criterion is first examined by making use of our previous data concerning an elevated straight girder bridge and, then, the validity and applicability of the new criterion are discussed in terms of a horizontally curved elevated girder bridge under bi-directional seismic accelerations. The details of the elevated straight girder bridge supported by 4 square thin-walled steel piers are shown elsewhere<sup>3)</sup>. As a numerical example to examine the validity and applicability of the new criterion, a 3-span horizontally curved elevated continuous girder steel bridge (Fig.2) supported by thin-walled circular steel piers is selected. The structural parameters of piers and a continuous girder are summarized in Tables 1 and 2. The girder that is assumed elastic is supported by fixed bearing at pier P2 and movable bearings at piers P1, P3 and P4. The movable bearing is free in the tangential direction of the curved girder axis. In order to express the local buckling behavior of bridge piers, the lower part of piers is discretized by nonlinear shell elements where the 3-surface cyclic plasticity model for material steel is implemented. The girder and upper part of piers are modeled by 3D Timoshenko beam elements. As input acceleration wave components, NS and EW components of variously magnified JRT waves are used.

**4. NUMERICAL RESULTS:** The accuracy of the new criterion is examined first by using the results previously obtained for straight girder bridge model<sup>3)</sup> whose stability was examined by the original criterion with Eq.(1). The relations between the magnification factor and response sway displacements of respective piers are shown in Fig.3. In this figure, it is also indicated for each pier whether or not instability state occurs during the seismic accelerations. It should be noted that the piers that once reached instability state frequently regain their stability under seismic accelerations. The original stability criterion predicts the overall instability of the bridge system under the seismic accelerations with the magnification factor  $\geq 0.62$  as shown in Fig. 3. On the other hand, the new criterion predicts the overall instability under seismic accelerations with factor  $\geq 0.59$ . This proves that the new criterion is reasonably conservative. Once the magnification factor on the JRT seismic acceleration exceeds the above threshold values, the response displacements of piers increase drastically.

Similar to the straight girder bridge model mentioned above, the curved girder bridge model (Fig. 2) is analyzed under bi-directional horizontal seismic accelerations; variously factored EW and NS components of JRT wave are simultaneously applied to the X-axis and the Y-axis directions, respectively, of global coordination system of bridge model. The relations between the magnification factor and response sway displacements of respective piers are shown in Fig.4. According to the new stability criterion, the curved girder bridge exhibits an overall instability under seismic accelerations with factor  $\geq 1.25$ . In view of the drastic increase in the response sway displacement in this range, the new criterion also appropriately predicts the overall instability of the overall curved girder bridge system. In the case of curved girder bridge, at least 3 piers must be unstable simultaneously for the system to be unstable. This is different from the straight girder bridge where overall instability occurs when P3 pier that supports the fixed bearing only becomes unstable. The similar movable directions of bearings in straight girder bridge may easily bring about the overall instability of the bridge system.

**5. CONCLUSIONS:** The proposed stability criterion of entire system is reasonably accurate and conservative, when applied to a straight elevated girder bridge. This criterion is also applicable to an elevated curved girder bridge.

**6.ACKNOWLEDGEMENT:** This research is partly supported by research fund of Japan Bridge Association.

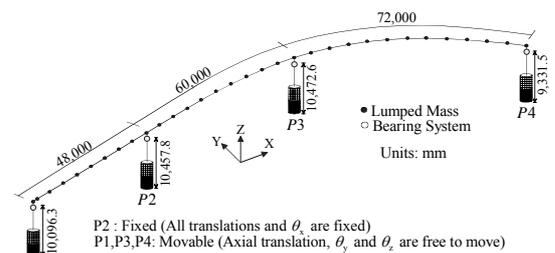
**REFERENCES:** 1) Japan Road Association, Specification for road bridges and commentaries, V. Seismic Design, Japan, 2002. 2) Hill, R.: *J. Mech. Phys. Solids*, Vol. 6, pp.236-249, 1958. 3) Goto, Y., et al.: *J. Struct. Eng.*, JSCE, Vol.57A, pp.490-499, 2011.

**Table 1** Geometrical properties of bridge piers

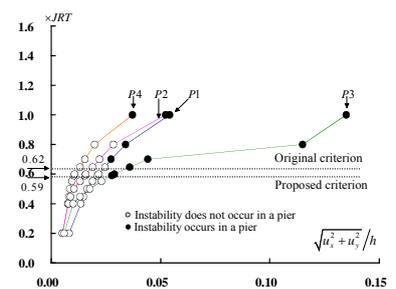
Pier	$h$ (mm)	$D$ (mm)	$t$ (mm)	$R_t$	$\lambda$	$P/\sigma_y A$
P1	9096.3	1620	26	0.08	0.402	0.0994
P2	9457.8	2610	41	0.08	0.259	0.0434
P3	9472.6	2310	37	0.08	0.294	0.0941
P4	9331.4	2050	32	0.08	0.326	0.0947

**Table 2** Structural parameters and mass of girders

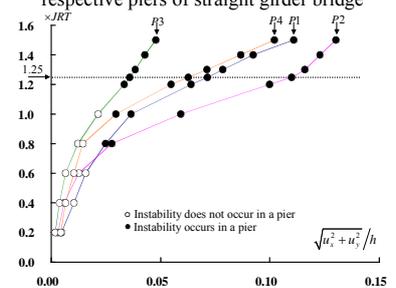
Span No	$A$ (m <sup>2</sup> )	$I_y$ (m <sup>4</sup> )	$I_z$ (m <sup>4</sup> )	$J_x$ (m <sup>4</sup> )	$M$ (ton)
S1	0.3762	0.3814	0.6507	0.2883	642.95
S2	0.6566	0.7135	0.9572	0.4998	549.12
S3	0.7624	0.9096	1.06056	0.5466	1106.18



**Fig. 2** Analytical model of horizontally curved elevated-girder bridge



**Fig. 3** Response sway displacements of respective piers of straight girder bridge



**Fig. 4** Response sway displacements of respective piers of curved girder bridge