Level set-based topology optimization of phononic crystals for sound barrier

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1. Introduction

Acoustic or elastic wave materials possessing a periodicity are called phononic crystals. Many researchers[1)] attempted to develop acoustic barriers by utilizing the band structure of the periodic materials. Since the capability of such a structure strongly depends on its profile, it is worthwhile to explore an effective shape. Therefore, the shape optimization method can be a practical tool for this task. Nishiwaki et al.[2)] have proposed a level set-based topology optimization method for periodic layers imbedded in an infinite photonic field. In Ref.[2)] the semi-infinite fields are approximately replaced to a finite region. In this paper a topology optimization method is developed for acoustic barriers based on the level set method. To represent the upper and lower acoustic half-planes, transmitting boundaries developed by Abe et al.[3)] are employed. Through a numerical example, the validity of the proposed method is discussed.

2. Wave transmission analysis

(1) Response analysis for incident wave

Let us consider a periodic layer consisting of acoustic scatterers as shown in **Fig.1**. In a unit cell of the layer, S_0, the equation of motion is given by

$$[\hat{\mathbf{K}}]\{\mathbf{u}\} = \{\mathbf{f}\}, \tag{1}$$

where $\{\mathbf{u}\}$ is the nodal sound pressure, $\{\mathbf{f}\}$ is the flux, $[\hat{\mathbf{K}}] = [\mathbf{K}] - \omega^2[\mathbf{M}]$, $[\mathbf{K}]$ and $[\mathbf{M}]$ are stiffness and mass matrices degenerated by the Bloch's theorem[4)]. In order to describe nodal fluxes at the lower and upper ends of S_0, unit cells S_1 and S_2 representing the half-planes are introduced. Impedance matrices $\mathbf{K}_{1U}$, $\mathbf{K}_{1D}$ and $\mathbf{K}_{2U}$ describing the relation between $\{\mathbf{u}\}$ and $\{\mathbf{f}\}$ at the boundaries are derived for S_1 and S_2[3)]. We then obtain an equation:

$$[\hat{\mathbf{K}}]\begin{Bmatrix} \mathbf{u}_B \\ \mathbf{u}_T \\ \mathbf{u}_M \\ \mathbf{u}_L \end{Bmatrix} = \begin{Bmatrix} -[\mathbf{K}_{1D}]\{\mathbf{u}_B\} + [\mathbf{K}_{1U} + \mathbf{K}_{1D}]\{\mathbf{u}^I\} \\ -[\mathbf{K}_{2U}]\{\mathbf{u}_T\} \\ 0 \\ 0 \end{Bmatrix}, \tag{2}$$

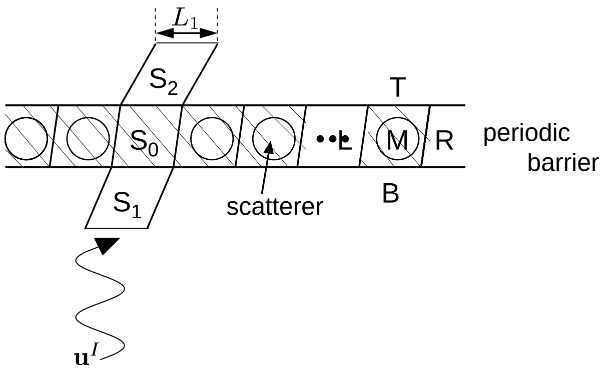

Fig. 1 Infinite acoustic field having a periodic barrier

where u_I is an incident wave. Arranging eq.(2) with respect to unknown nodal displacement, we can obtain the final equation for the response analysis.

(2) Energy transmittance

The energy transmittance E_r is defined as the energy ratio of the transmitting waves E_T to the incident waves E_I. E_T and E_I are given by

$$\begin{aligned} E_I &= \frac{\omega}{2}\mathrm{Im}([\mathbf{u}^{I*}][\mathbf{K}_{1D}]\{\mathbf{u}^I\}), \\ E_T &= \frac{\omega}{2}\mathrm{Im}([\mathbf{u}_T^*][\mathbf{K}_{2U}]\{\mathbf{u}_T\}), \end{aligned} \tag{3}$$

where $[\ ]^*$ stands for the conjugate transpose.

3. Toplogy optimization based on the leve lset method

(1) Level set function

The level set function ψ is defined by a signed distance function. Inside a scatterer the function has a positive value. In the topology optimization analysis once an advective speed $\mathbf{V}_n$ is determined by the sensitivity analysis, the level set function is updated based on the Hamilton-Jacobi equation as

$$\mathbf{V}_n \cdot \nabla\psi + \frac{\partial \psi}{\partial t} = 0. \tag{4}$$

(2) Design sensitivity analysis

The object function J is given by

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$$J = \int_{\Omega} E_r d\Omega + \lambda_+ (V - V_{\max}), \tag{5}$$

where Ω is a product space of intervals of the frequency ω and the incident angle θ. V and $V_{\max}$ are the current volume and an allowable volume limit, respectivelty. λ_+ is a Lagrange multiplier.

The sensitivity of the transmittance is expressed by

$$\begin{aligned} \Delta E_r &= \mathrm{Im}(-[\tilde{\mathbf{u}}_1^*][\Delta\hat{\mathbf{K}}]\{\mathbf{u}\} - [\tilde{\mathbf{u}}_2]^T[\Delta\bar{\hat{\mathbf{K}}}]\{\bar{\mathbf{u}}\}), \\ [\Delta\hat{\mathbf{K}}] &= \frac{\partial \mathbf{K}}{\partial \psi}\Delta\psi - \omega^2 \frac{\partial \mathbf{M}}{\partial \psi}\Delta\psi. \end{aligned} \tag{6}$$

$\{\tilde{\mathbf{u}}_1\}$ and $\{\tilde{\mathbf{u}}_2\}$ satisfy the following adjoint problems,

$$\begin{aligned} [\hat{\mathbf{K}}]^T\{\bar{\tilde{\mathbf{u}}}_1\} &= \frac{\omega}{2\bar{E}_I}[\tilde{\mathbf{K}}]^T\{\bar{\mathbf{u}}\}, \\ [\hat{\mathbf{K}}^*]\{\tilde{\mathbf{u}}_2\} &= \frac{\omega}{2\bar{E}_I}[\tilde{\mathbf{K}}]^T\{\mathbf{u}\}, \end{aligned} \tag{7}$$

where $[\tilde{\mathbf{K}}] = [\mathbf{B}]^T[\mathbf{K}_{2U}][\mathbf{B}]$, $[\mathbf{B}]$ is the boolean matrix extracting $\{\mathbf{u}_T\}$ from $\{\mathbf{u}\}$ and $[^-]$ is the conjugate.

4. Example

We consider an objective function evaluated at discrete points in Ω. Each scattering problem is given by a combination of two incident angles 0° and 45°, and five frequencies in the interval of $2.36 \leq \omega \leq 3.14$. Here the circular frequency ω is nondimensionalized by the speed of sound and the periodic length. The volume limit is given by 40% of the initial volume. The initial and the final shapes are shown in **Fig.2**. As shown in the figure, the initial shape is given by three circular obstacles, while the final shape has two scatterers. Time histories of the volume ratio and the objective function are shown in **Fig.3**. Although the convergence of the optimization required more than 5000 steps, after the convergence of the volume, the objective function decreases monotonically. **Figs.4** and **5** are showing the transmittance for $\theta = 0$ ° and 45°, respectively. From these figures, it is found that the optimal shape realizes the reduction of the transmittance successfully at the specified frequencies.

5. Conclusion

A topology optimization method has been developed for phononic crystals characterized by a periodic barrier. The lower and upper semi-infinite acoustic fields are represented by the aid of the impedance matrices. The topological change is achieved by the level set method. Through a numerical example, the feasibility of the method was proved.

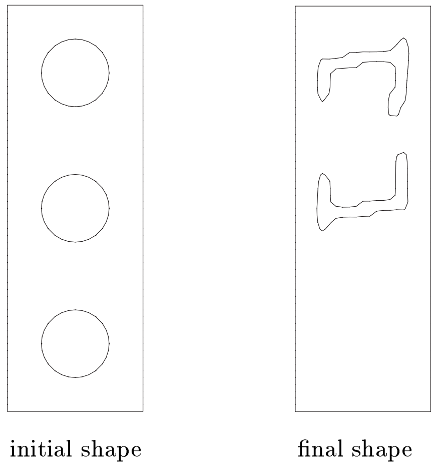


Fig. 2 Initial and final shapes

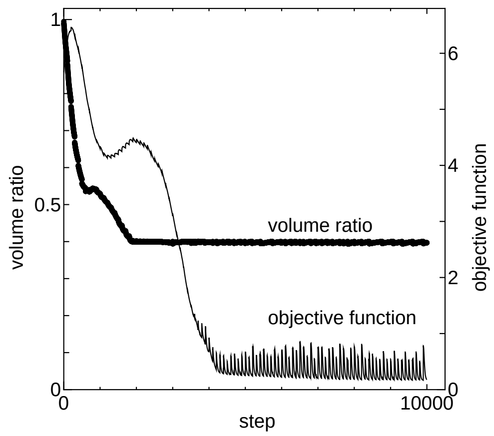
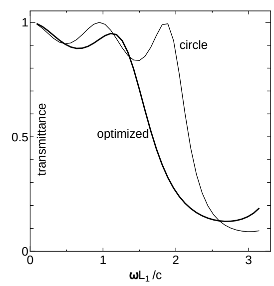


Fig. 3 Time histories of volume ratio and objective function



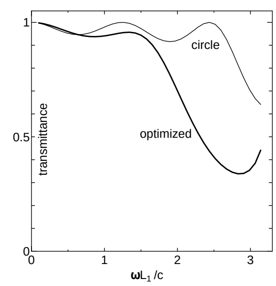


Fig. 4 Transmittance ($\theta = 0°$)

Fig. 5 Transmittance ($\theta = 45°$)

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