Nonlinear Elastic Buckling of CFRP Reinforced Steel Cylinders under Axial Compression

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1. Introduction

This paper is the sequential version of previous paper of JSCE 2009. On the previous paper, "the nonlinear numerical experiments have been carried out for CFRP laminated reinforced steel cylinders under axial compression and it has been predicted that depending upon the imperfections, the buckling mode as well as buckling loads differ. Also, it has been suggested that these variations are related with the results from the recently developed reduced stiffness method(RSM)[1]". Furthermore, on the proceedings of CICE2010, "it has been shown that the initial imperfection sensitivity is very dependent upon the lamination and the angle of fibre orientation, and buckling load carrying capacity increases when thin steel shell is reinforced with CFRP[2]". All of the above paper includes Reduced Stiffness (RS) analysis which predicts the lower bound but not in the case of larger imperfefection. To predict the lower bound for the larger imperfection sensitive buckling loads, a modified theory has been developed in this study and suggested that this modified RS theory has the potentiality to provide simple and reliable estimation for designing the larger imperfection sensitive buckling loads for CFRP reinforced steel cylinders under axial compression.

2. Method of Analysis

As shown in Fig. 1, the analytical model, having length L, radius of curvature R, shell thickness t, and axial load P is adopted here. The shell is considered simply supported and corresponding boundary condition is adopted as

$$w = 0,\ \ \partial^2 w/\partial x^2 = 0,\ \partial u/\partial x = 0,\ v = 0 \quad \text{at}\ \ x = 0, L \qquad (1)$$

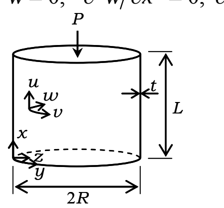


Fig.1: Shell geometry

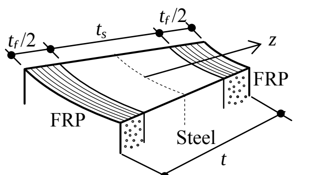


Fig.2: Lamination of steel and FRP's

As shown in Fig.2, the adopted geometrical parameters are L/R= 0.512 and R/t_s= 405[3]. On the other hand, t_f represents the thickness of carbon fibre and is taken as variable parameter in the present study, ranging thickness from 0 to t_s. Also, Young's moduli for steel, fibre and polymer are taken as E_s = 205GPa, E_F = 235GPa, E_P = 3.5GPa. Similarly, adopted Poisson's ratios are μ_s = 0.3, μ_F = 0.3 and μ_P = 0.34. To obtain the material constants and shear mudulus for composite geometry, Halpin-Tsai equation[4] is used.

3. Results and Discussions

Figure 3 shows the results of numerically experimented load-deflection curves for b=10 having imperfection amplitudes of 0.8, 2.4, 5.6, 6.4 and 7.2mm and shown that the buckling load carrying capacity is the highest for the reinforced condition having θ=90° for all the amplitudes.

Figure 4 shows the incremental buckling displacements at the buckling loads for the cases of an imperfection amplitude $w^0_{10,1}$ = 7.8mm. From this schematic figure, it can be observed that the axial wave length becomes sharper for the cases of reinforced conditions; this modified mode form reflects the influence of the CFRP reinforcement on the present numerical experimental models. Again, Fig. 5 shows the typical significant changes in mode at buckling as compared with the form of initial imperfection in the case of axial (y=0) and circumferential (x=L/2).

In Fig.6 an imperfection with b=10 is adopted since this mode results in the minimum nonlinear buckling loads. Whereas, the linearised critical buckling loads, and the nonlinear buckling loads for very small imperfections, exhibit considerable dependence upon the angle of fibre orientation, the buckling loads for large imperfections show remarkably load dependence upon the angle of fibre orientation.

Figure 7 is based upon the use of larger imperfections having a form (b,f) = (10,1) and are observed to produce buckling loads that are lower than P^*_{cm} associated with the mode $(i^*_{cm,} 1)$. But what is fascinating about the nonlinear results is that despite the shape of initial imperfection, ($w^0_{10,1}$), the incremental mode at buckling, at least when imperfection amplitudes are large, is dominated by wave form having considerably shortened circumferential and axial wave lengths. For the case of t_f = 2mm and θ = 90° shown in Fig. 5, for example, the incremental mode at buckling for the large imperfection $w^0_{10,1}$ = 7.8mm, has through a process of modal coupling reached localised shapes closer to that associated with (i,j) = (12, 2.26). The lowest RS critical load associated with larger imperfection is taken as modified RS load P^{**}_{cm} in this paper and it is no coincidence that this mode also happens to be the same as that for the lowest RS critical load associated with i^*_{cm} should then be that of P^*_c for $(i^*_{cm}, 2.26)$, which is depicted as P^{**}_{cm} ≈3.19MN in Fig. 7. Making use of this modified RS critical load (i.e. the lowest P^*_c associated with the mode $(i^*_{cm}, 1)$) can be seen to consistently provide extremely close approximations of the lower bounds to imperfection sensitive buckling loads.

Figure 8 represent the relationship between buckling load and the thickness of fibre t_f for angle of fibre orientation. Since, $P^{**}_{cm} < P_y$ by modified RS analysis suggests that the elastic buckling for moderately large imperfect shells in the use of practical civil engineering structures will occur first rather than material damage inducing collapse, P^{**}_{cm} provides consistent and reliable lower bounds over the entire range CFRP reinforcements considered.

In Fig.9, the linear buckling load P_{cm} becomes optimum at an angle of fibre orientation 35°. But as the results of nonlinear numerical experiments show, with angles of fibre orientation 20°, 35° and 70°, the imperfect shell buckling loads (P^N) and the RS critical loads are approximately the same for the imperfection amplitude $w^0_{b,1} = 2 \times t_s$.

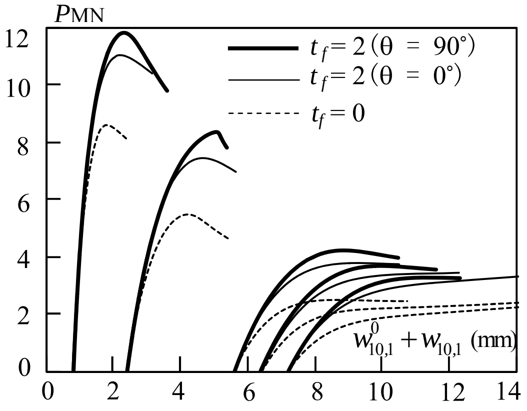
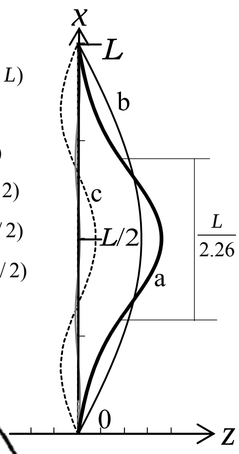


Fig.3: Load versus displacement curves



a: $\sum_i \sum_j^{J^0} w^d_{i,j} \sin(j\pi x / L)$

b: $\sum_i w^d_{i,1} \sin(\pi x / L)$

c: $\sum_i w^d_{i,3} \sin(3\pi x / L)$

Fig.4: Increamental displacement mode

d: $\sum_i \sum_j^{J^0} w^d_{i,j} \cos(iy / R) \sin(j\pi / 2)$

e: $\sum_j w^d_{10,j} \cos(10y / R) \sin(j\pi / 2)$

f: $\sum_j w^d_{20,j} \cos(20y / R) \sin(j\pi / 2)$

$t_f = 2$, $\theta = 90^\circ$, $w^0_{10,1} = 7.8$mm

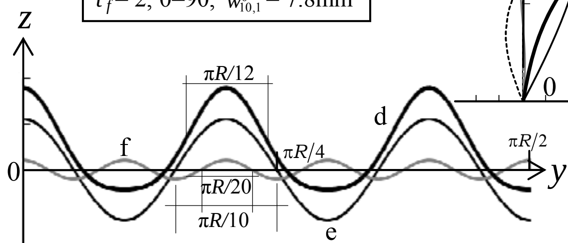


Fig. 5: Increamental wave forms at buckling points

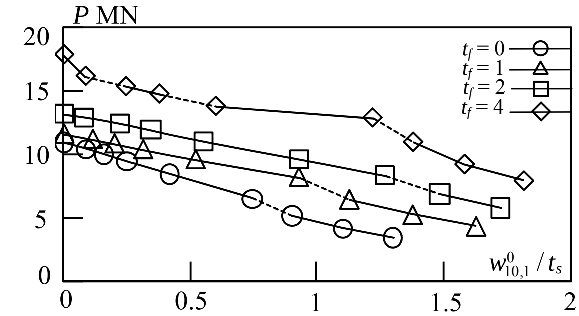


Fig.6: Sensitivity of buckling load with imperfection amplitude for $\theta = 90^\circ$

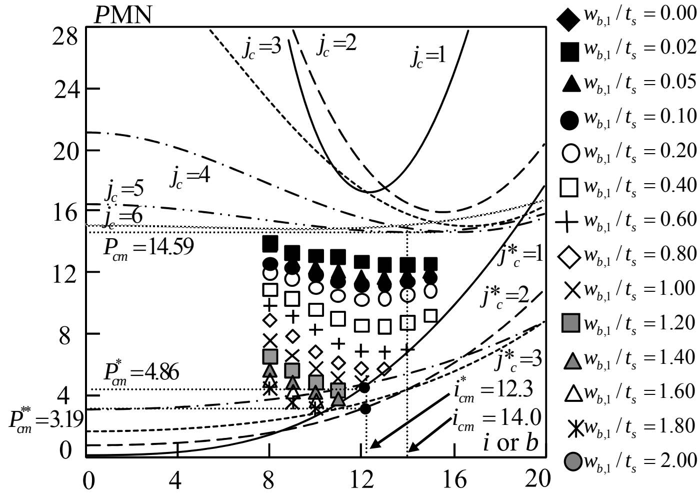


Fig.7: Load spectra for $t_f = 2$, $\theta = 90^\circ$

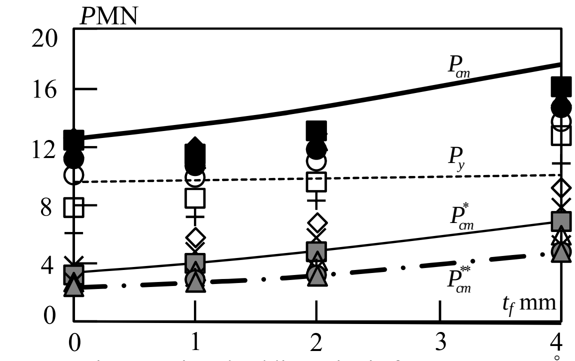


Fig.8: Various buckling criteria for $b = 10$, $\theta = 90^\circ$

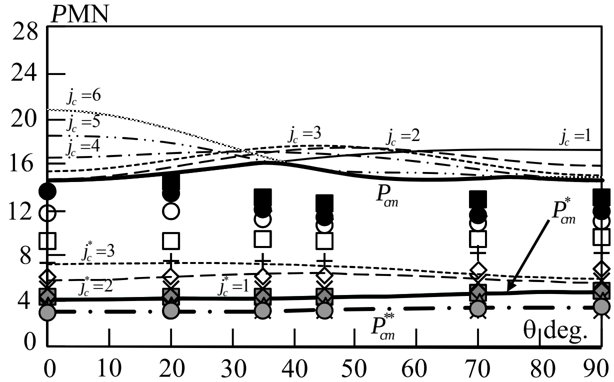


Fig.9: Effects of fibre orientation for $t_f = 2$

Table 1 Critical loads associated with axual half-wave and circumferential full wave numbers

Angle of fibre orientation	t_f mm	Linear			RS			Modified RS		
		i_{cm}	j_{cm}	P_{cm} MN	i^*_{cm}	j^*_{cm}	P^*_{cm} MN	$i^{**}_{cm} \equiv i^*_{cm}$	j^{**}_{cm}	P^{**}_{cm} MN
	0	17.20	4	12.47	13.70	1	3.35	13.70	2	2.35
θ = 0°	1	17.20	2	13.44	13.70	1	3.78	13.70	2	2.89
	2	13.60	1	14.59	13.60	1	4.24	13.60	2	3.61
	4	13.40	1	17.52	13.40	1	5.61	13.40	1	5.14
θ = 90°	1	16.80	4	13.43	13.00	1	4.03	13.00	2	2.70
	2	14.00	5	14.59	12.30	1	4.86	12.30	2	3.19
	4	13.10	5	17.56	11.10	1	6.88	11.10	1	4.76

4. Conclusion

Nonlinear numerical experiment, linear, RS analysis and modified RS Analysis have been carried out for CFRP reinforced steel cylinders under axial compression. It is observed that depending upon the form and magnitude of imperfections, the buckling as well as buckling load carrying capacities differs. The buckling load carrying capacity is high for small imperfection amplitudes but is strongly dependent upon the thickness of the adopted reinforcement. Furthermore, the modified reduced stiffness theory provides the reliable estimation for designing the larger imperfection sensitive buckling loads predicting the lower bound for CFRP reinforced steel cylinders under axial compression.

References

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2) S.Yamada, K.K. Bhetwal, M. Yanagida and Y. Matsumoto: Effects of FRP reinforcements on the buckling and reduced stiffness criteria of compressed steel cylinders, *Proceedings of The 5th International Conference on FRP Composites in Civil Engineering*, CICE, pp.925-928, 2010.
3) Yamada, S. and Croll, J.G.A. (1999) “Contributions to understanding the behavior of axially compressed cylinders”, *J. Applied Mechanics*, ASME,Vol. 66, 299-309.
4) Jones, R.M.(1999), *Mechanics of Composite Materials*.