Efficient large scale FE dynamic analysis using model order reduction via Krylov subspace

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1. Introduction

Recently, a model order reduction (MOR) for spatially discritized time differential equations has been introduced by using a projection into the Krylov subspace(KS). In this study, the conventional FEM is utilized for the spatial discritization. Arnoldi algorithm (AR), which is one of the scheme to generate Krylov basis vectors, is selected for its robustness and efficiency.

In the basis vectors generation process by using AR, linear equation solver with the original degree of freedoms is required. To apply KS-MOR into large scale problems, the iterative solver, such as conjugate gradient method, is strongly desired. Then, performance of iterative solver for generating basis vectors has been checked with over 1 million degree of freedoms problem.

2. FE governing equation for structure dynamics for prescribed displacement problems

The matrix form governing equations after discretize with the consideration of boundary condition can be derives in dynamic equation form as follows,

$$\textstyle\sum_N : \{\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{D}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \alpha(t)\mathbf{F}, \tag{1}$$

where $\mathbf{M} \in \Re^{N\times N}$, $\mathbf{D} \in \Re^{N\times N}$, $\mathbf{K} \in \Re^{N\times N}$ indicate the mass, damping and stiffness matrix respectively, and $\ddot{\mathbf{u}}(t) \in \Re^N, \dot{\mathbf{u}}(t) \in \Re^N, \mathbf{u}(t) \in \Re^N$ are acceleration, velocity and displacement. $\mathbf{F} \in \Re^{N\times l}$ is block of force matrix. The damping matrix $\mathbf{D}$ is calculated using Rayleigh damping, $\mathbf{D} = \alpha_{\mathrm{R}}\mathbf{M} + \beta_{\mathrm{R}}\mathbf{K}$. Here, $\alpha(t)$ is a scalar load function of time.

By renumbering unknown vector components, a block system equation can be written as

$$\begin{bmatrix} \mathbf{M}_{\overline{\mathrm{F}}} & \mathbf{M}_{\overline{\mathrm{CF}}} \\ \mathbf{M}_{\overline{\mathrm{CF}}} & \mathbf{M}_{\overline{\mathrm{C}}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_{\overline{\mathrm{F}}} \\ \ddot{\overline{\mathbf{u}}}_{\overline{\mathrm{C}}} \end{Bmatrix} + \begin{bmatrix} \mathbf{D}_{\overline{\mathrm{F}}} & \mathbf{D}_{\overline{\mathrm{CF}}} \\ \mathbf{D}_{\overline{\mathrm{CF}}} & \mathbf{D}_{\overline{\mathrm{C}}} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}_{\overline{\mathrm{F}}} \\ \dot{\overline{\mathbf{u}}}_{\overline{\mathrm{C}}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{\overline{\mathrm{F}}} & \mathbf{K}_{\overline{\mathrm{CF}}} \\ \mathbf{K}_{\overline{\mathrm{CF}}} & \mathbf{K}_{\overline{\mathrm{C}}} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_{\overline{\mathrm{F}}} \\ \overline{\mathbf{u}}_{\overline{\mathrm{C}}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_{\overline{\mathrm{F}}} \\ \mathbf{0} \end{Bmatrix}, \tag{2}$$

where the subscript $\overline{\mathrm{C}}$ indicates values related to constraint nodes and the subscript $\overline{\mathrm{F}}$ indicates unknown values. We assume that $\ddot{\overline{\mathbf{u}}}_{\overline{\mathrm{C}}}, \dot{\overline{\mathbf{u}}}_{\overline{\mathrm{C}}}, \overline{\mathbf{u}}_{\overline{\mathrm{C}}}$ are prescribed before FEA. Note that the nodal points given a prescribed displacement, velocity and acceleration cannot take an external force vector. That is, $\mathbf{f}_{\overline{\mathrm{C}}} = 0$ at any time. It is assumed that the prescribed condition can be given by $\mathbf{f}_{\mathbf{M}} = -\mathbf{M}_{\overline{\mathrm{CF}}}\ddot{\overline{\mathbf{u}}}_{\overline{\mathrm{C}}}$, $\mathbf{f}_{\mathbf{D}} = -\mathbf{D}_{\overline{\mathrm{CF}}}\dot{\overline{\mathbf{u}}}_{\overline{\mathrm{C}}}$ and $\mathbf{f}_{\mathbf{K}} = -\mathbf{K}_{\overline{\mathrm{CF}}}\mathbf{u}_{\overline{\mathrm{C}}}$. The equation(2) should only be solved for $\ddot{\mathbf{u}}_{\overline{\mathrm{F}}}$, $\dot{\mathbf{u}}_{\overline{\mathrm{F}}}$ and $\mathbf{u}_{\overline{\mathrm{F}}}$. Therefore, these equations can equivalently written as

$$\mathbf{M}_{\overline{\mathrm{F}}}\ddot{\mathbf{u}}_{\overline{\mathrm{F}}} + \mathbf{D}_{\overline{\mathrm{F}}}\dot{\mathbf{u}}_{\overline{\mathrm{F}}} + \mathbf{K}_{\overline{\mathrm{F}}}\mathbf{u}_{\overline{\mathrm{F}}} = \mathbf{F}\begin{bmatrix} \alpha_{\overline{\mathrm{F}}}(t) \\ \alpha_{\mathbf{M}}(t) \\ \alpha_{\mathbf{D}}(t) \\ \alpha_{\mathbf{K}}(t) \end{bmatrix}; \quad \mathbf{F} = \left[\mathbf{f}_{\overline{\mathrm{F}}}\ \ \mathbf{f}_{\mathbf{M}}\ \ \mathbf{f}_{\mathbf{D}}\ \ \mathbf{f}_{\mathbf{K}}\right]. \tag{3}$$

Here, the scalar load function are given by

$$\begin{aligned} &\alpha_{\mathbf{K}} = \sin(2\pi ft), \alpha_{\mathbf{D}} = 2\pi f\cos(2\pi ft), \\ &\alpha_{\mathbf{M}} = -4\pi^2 f^2 \sin(2\pi ft). \end{aligned} \tag{4}$$

3. Generation of basis vectors using BSOAR and dimension reduction

To manage multiple inputs problems, such as eqn.(3), Block Second Order AR (BSOAR) is utilized instead of AR. The flowchart of BSOAR is summarized in **Algorithm 1**. For prescribed displacement problem defined in section 2, $\mathbf{M}_{\overline{\mathrm{F}}}$, $\mathbf{D}_{\overline{\mathrm{F}}}$ and $\mathbf{K}_{\overline{\mathrm{F}}}$ are substituted into all the equation in Algorithm1 instead of $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$. Note again that this process is just to generate basis vectors in Krylov subspace. After this process, a simple projection with these vectors has been conducted as shown in **Figure 1**. Once the equivalent small system equations can be generated, Newmark-β method can be applied to evaluate the time history. The real space solution can be easily represented by inverse projection using the same basis vectors.

Keywords: *Large scale computation, Krylov subspace, Iterative solver, Structural Dynamics*,

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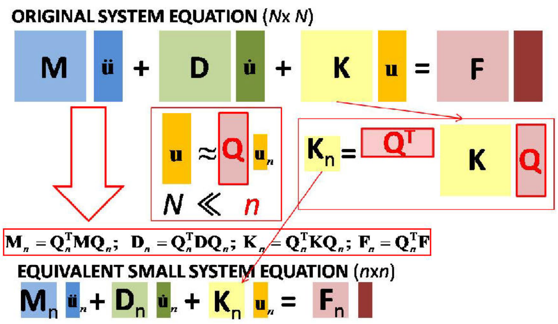

Figure 1. KS-MOR concept

Algorithm 1: BSOAR procedure [1)]

$\mathbf{Q}_1 = [q_1, q_2, \ldots, q_l]$
$p_0 = 0$
for $j = 1,2,\ldots,m (= m_1 \times l)$ do
  $r = \mathbf{DK}^{-1} q_j + \mathbf{MK}^{-1} p_j$
  $s = q_j$
  for $i = 1,2,\ldots, j+l-1$ do
  $t_{ij} = q_i^{\mathrm{T}} r$
    $r := r - q_j\, t_{ij}$
    $s := s - p_j\, t_{ij}$
  end for
    $t_{j+l\,j} = \|r\|_2$
  if $t_{j+l\,j} = 0$, breakdown
  else
   $q_{j+l} = r / t_{j+l\,j}$
   $p_{j+l} = s / t_{j+l\,j}$
  end if
 end for

4. Numerical example

A laminated rubber bearing bridge model, which is shown in **Figure 2,** was simulated by using KS-MOR. The material is assumed by isotropic elasticity with material parameters shown in **Table 1**. The FE model has 392,036 quadratic Hexa elements, and 1,457,184 DOFs. The coefficients for Newmark-β are fixed by $\beta = 0.25$ and $\delta = 0.5$. The Rayleigh damping coefficient are assumed as $\alpha_R = 0.1$ and $\beta_R = 0.1$. The model is prescribed displacement at maximum 1mm with scalar loading function as in equation 4.

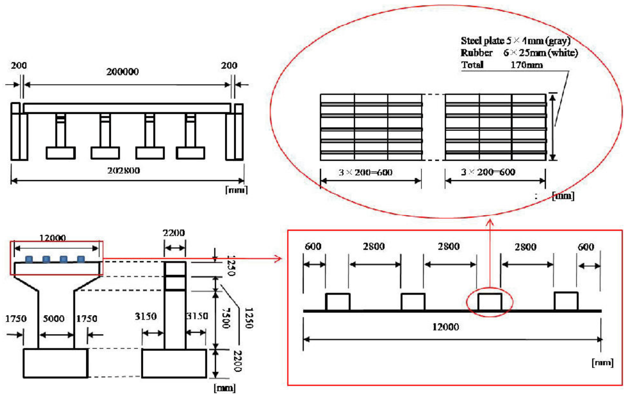

Figure 2. Dimension of bridge model

Table 1. bridge model's properties

	young's module [Gpa]	poisson's ratio [-]	density [kg/m3]
steel	210	0.30000	7874
rubber	0.01	0.49999	910
concrete	30	0.16666	2300

By using the iterative solver, over 1 million DOFs problems can be computed by using our workstation with 12GB memory. In the **Figure 3**, the stress distribution is compared between the FEM reference solution and KS-MOR. The current KS-MOR solution use only 10 basis vectors. It is recommended to increase the number of basis vectors and to check the convergence tolerance of the iterative solver (conjugate gradient method).

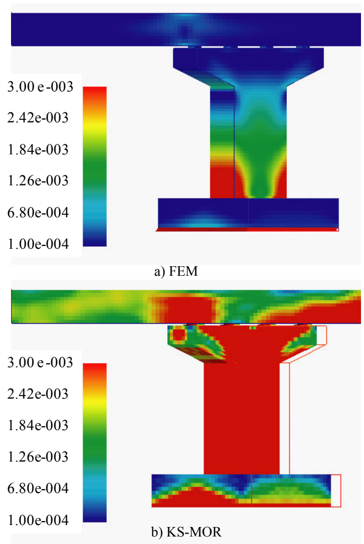

Figure 3. Stress distribution of bridge model

5. Conclusion

To apply KS-MOR into large scale dynamic FE problems, the iterative solver has been introduced, and the applicability has been investigated. The current solutions are not enough to discuss the performance of KS-MOR with iterative solver, and some more information may be included at our presentation in conference. In the future, we will focus on how to improve the accuracy and efficiency.

Reference

1)Y.Lin, L.Bao, Moel Order reduction of large –scale second order MIMO dynamical systems via block second order Arnoldi method, Int. Journal of Comp. Math., Vol 84, No 7, July 2007, 1003-1019