State Updating and Forecasting of Multiple Hydrologic Models using Sequential Monte Carlo Methods

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1. Introduction

To make forecasts of near future is one of key issues in hydrology. The requirement is to provide predictions at the lead time of interest with minimum uncertainty, and data assimilation is needed to improve the predictions and reduce the forecast uncertainty (Beven, 2009).

Among data assimilation techniques, sequential Monte Carlo (SMC) methods are Bayesian learning process in which the propagation of all uncertainties is carried out by a suitable selection of randomly generated particles without any assumptions about nature of the distributions. SMC has the advantage of being applicable to non-linear, non-Gaussian state-space models. Over the last few years, the application of these powerful and versatile methods has been increasing, e. g., pattern recognition, weather forecasting, bioinformatics, etc.

The objective of this study is to show the applicability of SMC methods for state updating and forecasting of multiple hydrologic models from conceptual one to process-based and spatially-distributed ones. In this study, three hydrologic models, Storage Function (SF) model, Water and Energy transfer Processes (WEP) model (Jia et al, 2009) and Kinematic Wave Method for Surface and Subsurface runoff (KWMSS) (Ichikawa et al, 2001) are implemented for the middle-sized Japanese catchment. Several variants of SMC methods are applied and compared for the improvement of forecasting capabilities.

2. Methodology

Consider a generic dynamic state-space model which can be described as follows:

$$x_t = f(x_{t-1}, \theta, u_t) + \omega_t \qquad \omega_t \sim N(0, W_t) \tag{1}$$

$$y_t = h(x_t, \theta) + \nu_t \qquad \nu_t \sim N(0, V_t) \tag{2}$$

Where $x_t \in \Re^{n_x}$ is the n_x dimensional vector denoting the system state at time t. The operator $f : \Re^{n_x} \to \Re^{n_x}$ and $h : \Re^{n_x} \to \Re^{n_x}$ express the system transition in response to the forcing data u_t, parameters θ. ω_t and ν_t represent the model and the measurement error, respectively.

In the Bayesian recursive estimation, if the system and measurement models are non-linear and non-Gaussian, it is not possible to construct the PDF of the current state x_t given all the measurement analytically. SMC is based on point mass ("particle") representations of probability densities with associated weights (Arulampalam et al., 2002).

$$p(x_t \mid Y_t) \approx \sum_{i=1}^{N} w_t^i \delta(x_t - x_t^i) \tag{3}$$

where x_t^i, w_t^i denote the i th particle and its weight, respectively, and $\delta()$ denotes the Direc delta function.

Since it is usually impossible to sample from the true posterior PDF, an alternative is to sample from a proposal distribution, also called importance density, denoted by $q(x_t \mid y_t)$. The recursive weight updating could be derived as follows:

$$w_t^i \propto w_{t-1}^i \frac{p(z_t \mid x_t^i) p(x_t^i \mid x_{t-1}^i)}{q(x_t^i \mid x_{t-1}^i, y_t)} \tag{4}$$

Several variants of SMC methods such as Sequential Importance Resampling (SIR), Auxiliary SIR (ASIR) and Regularized Particle Filter (RPF) have been developed to overcome the degeneracy phenomenon, selection of importance density and sample impoverishment.

Keywords: hydrologic forecasting, data assimilation, sequential Monte Carlo, particle filter
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For the unknown parameters, the concept of "artificial evolution" can be applied. That means, θ is replaced at each time adding an independent, zero-mean normal increment to the parameter as follows:

$$\theta_t = \theta_{t-1} + \zeta_t \qquad \zeta_t \sim N(0, R_t) \tag{5}$$

In this concept, the distribution of parameters moves at each time to reproduce the observed data.

3. Preliminary results and discussion

Three different hydrologic models were applied to the Katsura river catchment to forecast the stream discharge through SMC methods. This catchment is located in Kyoto, Japan and covers an area of 1,100 km^2 (887 km^2 at the Katsura station). Hourly observed precipitation and discharge data were used. In preliminary results, SF model was simulated by three different SMC methods; SIR, ASIR and RPF with MCMC move step. 1,000 particles were used for each simulation. Results show that three SMC methods could enhance forecasting capabilities of SF model compared to deterministic prediction (Figure 1). Figure 2 shows traces of storage with probabilistic width. Uncertainty of parameters was analyzed through several approaches including artificial evolution. For example, Figure 3 illustrates distribution of parameters at each time. Simulation results of distributed hydrologic models, WEP and KWMSS, would be also provided and analyzed together in the session.

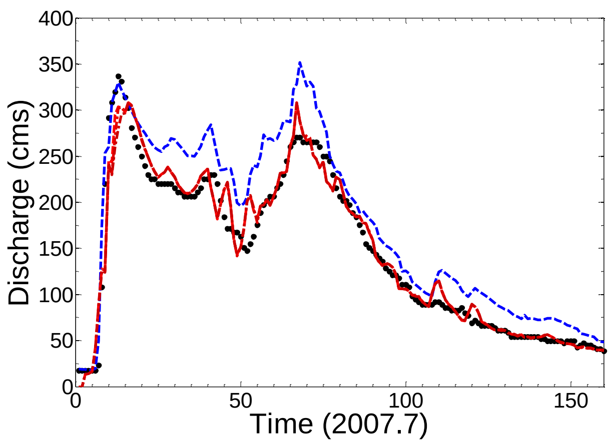

Figure 1. Hourly discharge hydrograph from 12 to 18 July 2007. Red lines represent 3-hr-lead forecast using three SMC methods, respectively. Blue dashed line represents deterministic prediction without data assimilation. Blue dots denote observed values.

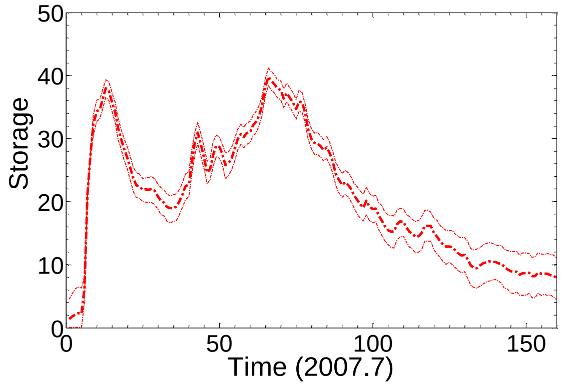

Figure 2. Hourly traces of storage from 12 to 18 July 2007. Red solid and dashed lines represent mean and 95% confidential intervals of storage using RPF with MCMC move step, respectively.

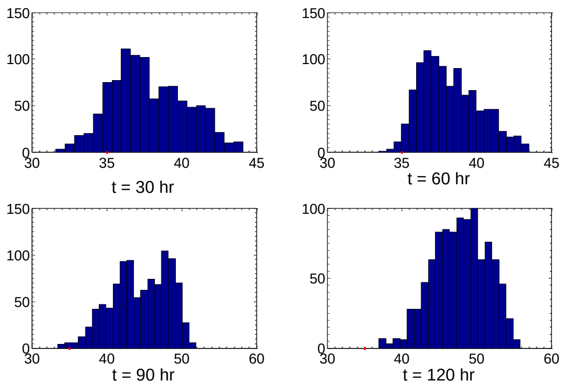

Figure 3. Time evolution of SF model parameter k distribution at four different time segments using ASIR

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