A new equation for equivalent period and damping of SDOF systems subjected to near source earthquakes

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1. Introduction

One of the most promising design methods proposed in the previous years is based on displacement rather than forces. In this method, the structure is simplified into an equivalent SDOF having equivalent properties namely period and damping. Although extensive research has been carried out to develop equations for the calculation of these properties, further research is needed to include the effects of the site conditions (e.g. soil and type of seismic motion), and also the inherent structural properties (e.g. natural period). In this work, these factors are included and equations for equivalent properties are derived for the near source earthquake specified by the Japanese Seismic Design Specifications for Highway Bridges. The validation and verification of the results obtained with the proposed equations is also presented.

2. Fundamentals

2.1 Applications of equivalent period and damping

One of the most direct applications of the equivalent properties is in the seismic displacement-based design. In this method, the maximum displacement demands, u, can be obtained straightforwardly from the design acceleration spectra, S_a, given by the design guidelines with the following equation

$$u = S_d(T_{eq}, \xi_{eq}) = \left(\frac{T_{eq}}{2\pi}\right)^2 \left(\frac{1}{\eta}\right) S_a(T_{eq}, \xi = 5\%) \tag{1}$$

where η is the factor for damping reduction defined in the Japanese specifications, T_{eq} and ξ_{eq} are the equivalent period and damping respectively.

2.2 Equivalent linearization.

The objective is to find expressions for T_{eq} and ξ_{eq} to be substituted in equation (1). The resulting equations for these *equivalent properties* shall depend on know quantities. To this end, equivalent linearization concepts will be used as explain below.

Consider the motion of an inelastic single degree of freedom (SDOF) system given by

$$m\ddot{x} + c\dot{x} + R(x) = -\ddot{z}_g \tag{2}$$

where m and c are the mass and damping of the system respectively, R is the restoring force, $\ddot{z}_g$ is the ground excitation, x is the displacement and over dots indicate time derivatives. To model the inelastic behavior, the modified Clough hysteretic model was adopted in this study.

The maximum inelastic displacement can be calculated by nonlinear analysis of equation (2) or can be approximately estimated by the response of an equivalent linear oscillator with motion equation, obtained after simplification of equation (2) considering $\omega_{eq} = \sqrt{k_{eq}/m}$, given by

$$\ddot{x} + \left(\frac{4\pi}{T_{eq}}\right)\xi_{eq}\dot{x} + \left(\frac{2\pi}{T_{eq}}\right)^2 x = -\ddot{z}_g \tag{3}$$

2.3 Response spectrum compatible accelerograms.

In order to obtain the input ground excitation for equations (2) and (3), a family of 30 earthquakes compatible with the acceleration spectra for near source earthquakes, Level 2 -type 2- (earthquake S_{II0}), specified in the Japanese Seismic Design Specifications for Highway Bridges, were generated for each type of soil following the next steps: a) the power spectra is derived from the acceleration design spectra, b) it is converted to nonstationary wave by multiplying it with a deterministic time modulating function, c) the resulting wave is iteratively adjusted to match the spectra with the Fast Fourier Transform (FFT) and the inverse FFT.

Keywords: equivalent linearization, equivalent period, equivalent damping, displacement spectra

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3. Methodology

1. Calculate the difference (ε) between the inelastic displacement and the shifted elastic displacement for all earthquakes, ductility values, (μ), and range of periods T_0 considered by

$$\varepsilon(\xi_{eq}, T_{eq}) = \frac{SD_e(T_{eq}(T_0,\mu), \xi_{eq})}{SD_{nl}(\mu, T_0, \xi_0)} - 1 \tag{4}$$

2. Compute the root-mean-square error, ($\bar{\varepsilon}$), for the total number of earthquakes (N), for each ductility and period with

$$\bar{\varepsilon}(\varepsilon_{eq}, T_{eq}) = \sqrt{\sum_{i=1}^{N} \frac{\varepsilon_i^{\ 2}}{N}} \tag{5}$$

3. Find the optimum combination of ξ_{eq} and T_{eq}, which gives the minimum error $\bar{\varepsilon}$. This can be done by forming a three dimensional region ξ_{eq} - T_{eq}/T_0 - $\bar{\varepsilon}$. If the equivalent properties are set in the horizontal plane and the error in the vertical axis, the pair of ξ_{eq} and T_{eq} which gives a value zero, or close to zero, for the ordinate $\bar{\varepsilon}$ shall be the optimum.

4. Perform regression analyses to fit a curve which closely match the equivalent properties for all range of periods. Finally, validate the results by statistical analysis.

4. Results

In previous studies, the equations for the equivalent properties include a linear term for the period, in this work, a nonlinear term for the period is newly introduced as below

$$T_{eq} = T_0[1 + aT_0^{\ b}(\mu^c - 1)] \tag{6}$$

$$\xi_{eq} = \xi_0 + dT_0^{\ f}(\mu - 1) \tag{7}$$

where a, b, c, d, and f are constants calculated for each type of soil as shown in Table I

Soil	A	b	c	d	f
I	0.3709	-0.4594	0.4785	0.03291	-0.0372
II	0.2376	0.1883	0.7982	0.0467	-0.0922
III	0.2534	-0.1401	0.7902	0.0418	-0.0712

Table 1. Constants for the equivalent properties calculation

4. Verifications

To evaluate the accuracy of the values given by equations 6 and 7, the mean approximate to exact displacement ratios, and the standard deviation of the error were calculated for each type of soil and ductility value. The results are presented in Figures 1 and 2.

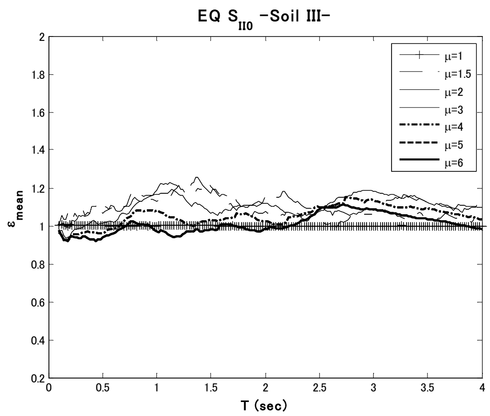

Figure 1. Mean approximate to exact displacement ratios

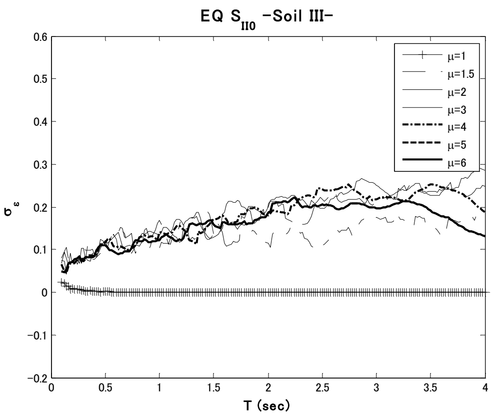

Figure 2. Standar deviation of relative errors

4. Conclusions

New equations to calculate the equivalent period and damping of SDOF systems subjected to the near source earthquakes specified in the Japanese Seismic Design Specifications for Highway Bridges were proposed. According with the verification analysis, the error and dispersion obtained with these equations are maintained under tolerance limits.