LARGE DEFORMATION ANALYSIS OF MEMBRANE STRUCTURES CONSIDERING SMALL COMPRESSIVE STIFFNESS

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1. INTRODUCTION

Membrane structures are widely utilized in the world because of large span, lightweight material, short construction cycle and novel appearance. We are proposing a method using cable units to analyze membrane structures. The cable unit in the tense state has the same stiffness as that of the uniform strain element generally used to model membrane structures, and the unit can easily express compression-free behaviors of a large-scale of membrane structure in the construction. The paper shows that the cable unit can also take small compressive stiffness in it. The uniform strain element is using small value of Young's modulus as the compressive stiffness. In real membrane structures, however, the compressive resistance results from the bending stiffness. The study adds the compressive stiffness derived from the bending stiffness of the membrane to the cable unit and computes the effect of stiffness to the large deformation phenomena of the membrane structures.

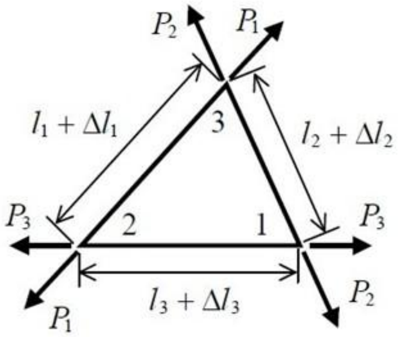

Figure1:Uniform strain element

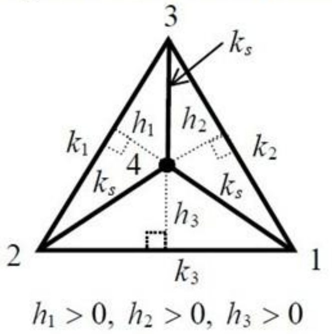

Figure2:Cable unit element

2. CABLE UNITS TO APPROXIMATE MEMBRANE

The triangular element with uniform strain is extensively applied in the analysis of membrane structure. The element deformation has three independent degrees of freedom, namely the elongations of the three element sides shown by Figure1.The stiffness equation (1) of the uniform strain element is derived from the elongations and the element forces.

$$\begin{Bmatrix}P_1\\P_2\\P_3\end{Bmatrix}=\frac{D}{4A}\begin{bmatrix}e_1^2+\mu l_1^2 & e_1e_2-\mu l_1l_2 & e_3e_1-\mu l_3l_1\\ & e_2^2+\mu l_2^2 & e_2e_3-\mu l_2l_3\\ sym. & & e_3^2+\mu l_3^2\end{bmatrix}\begin{Bmatrix}\Delta l_1\\\Delta l_2\\\Delta l_3\end{Bmatrix}=\begin{bmatrix}K_{11} & K_{12} & K_{13}\\ & K_{22} & K_{23}\\ sym. & & K_{33}\end{bmatrix}\begin{Bmatrix}\Delta l_1\\\Delta l_2\\\Delta l_3\end{Bmatrix},D=\frac{Et}{1-\nu^2},\mu=\frac{1-\nu}{2} \quad \text{(1a,b,c)}$$

Where t: the membrane thickness, A: the stressless element area, E: the Young's modulus, ν: the Poisson's ratio, l_n : the length of stressless element's side n, e_n : the distance between the apex n and the orthocenter.

On the basis of keeping the same behavior with uniform strain element, the study proposes the cable unit shown by Figure2. The cable unit element consists of six members. The three members calling main member constitute the sides of cable unit element whose shape is the same as the stress element. The other three ones calling subsidiary member connect the three apexes with the additional node 4. The stiffness equation (2) of the cable unit element can be deduced by appropriately determining the stiffness of six members and the position of additional node. In the replacement, the strain energy of cable unit is equalized to that of the uniform strain element. Therefore, the stiffness equation of the cable unit element is equal to that of the uniform strain element.

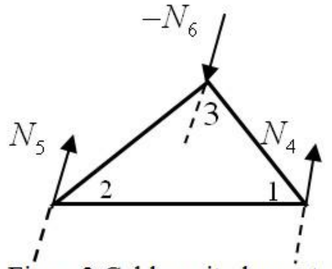

Figure3:Cable unit element with Poisson ratio 1/3

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$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} k_1 + k_s/(Fh_1^2) & k_s/(Fh_1h_2) & k_s/(Fh_3h_1) \\ & k_2 + k_s/(Fh_2^2) & k_s/(Fh_2h_3) \\ sym. & & k_3 + k_s/(Fh_3^2) \end{bmatrix} \begin{Bmatrix} \Delta l_1 \\ \Delta l_2 \\ \Delta l_3 \end{Bmatrix}, F = 2\left[\frac{1}{h_1^2} + \frac{1}{h_2^2} + \frac{1}{h_3^2} + \frac{h_1 Cos\theta_1 + h_2 Cos\theta_2 + h_3 Cos\theta_3}{h_1h_2h_3}\right] \quad (2a,b)$$

Where k_n: the stiffness of the main member n, k_s: the stiffness of the three subsidiary member, h_n: the length of the perpendicular from the additional node to side n , θ_n: the interior angle at apex n.

The position of additional node expressed by the h_n can be determined by the stiffness of the uniform strain element as follows,

$$h_1 = \frac{K_{23}}{K_{sT}}, \quad h_2 = \frac{K_{13}}{K_{sT}}, \quad h_3 = \frac{K_{12}}{K_{sT}}, \quad K_{sT} = \frac{K_{23}}{a_1} + \frac{K_{13}}{a_2} + \frac{K_{12}}{a_3} \quad (3a,b,c,d)$$

Where a_n : the length of the perpendicular from the apex n to side n.

The stiffness of each subsidiary member

$$k_s = 2\left(\frac{K_{23}K_{13}}{K_{12}} + \frac{K_{12}K_{23}}{K_{13}} + \frac{K_{13}K_{12}}{K_{23}} + K_{23}cos\theta_1 + K_{13}cos\theta_2 + K_{12}cos\theta_3\right) \quad (4)$$

The stiffness of each main member

$$k_n = K_{nn} - k_s/(Fh_n^2), \ (n=1,2,3) \quad (5)$$

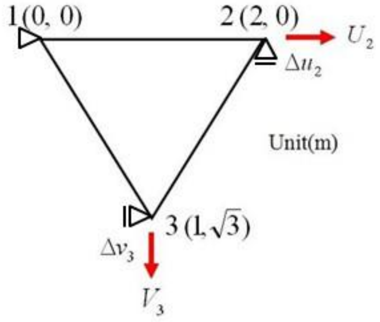

Figure4:Cable unit element of equilateral triangle

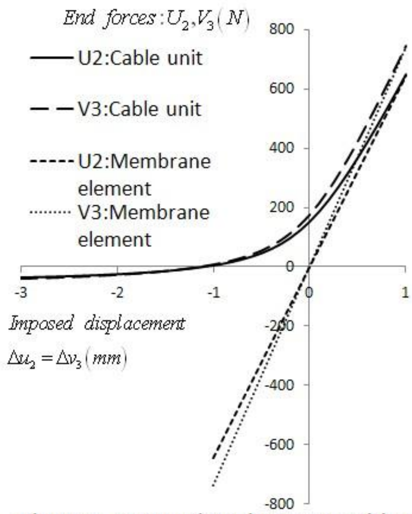

Figure5: Comparison between cable unit and membrane element

The study analyzes the large deformation of membrane structures with the Poisson's ratio 1/3 membrane materials. The analysis becomes easy and stable because it does not need to solve the position of additional node during the deformation of element. Furthermore, there is no subsidiary member in the equilateral triangle element whose Poisson's ratio is 1/3. In the other triangle elements, the additional node is infinite far so the subsidiary members are parallel like the Figure3.

3. BEHAVIORS OF CABLE UNIT

The cable unit has almost same behavior as the uniform strain element in the tense state and the small compressive stiffness can be added to the unit. Considering these behaviors, the study proposes the equation (6) to calculate the axial force of cable unit. Figure4 and 5 demonstrate the confirmatory example. The equilateral triangle element in Figure4 has the side length of 2m and the physical quantities of the membrane material are the elongation stiffness of Et=882kN/m, the weight per unit area of w=9.8N/m^2 and the Poisson's ratio of ν=1/3.

$$N = \frac{k}{2}\Delta l + (N_0 + N_c)\sqrt{1 + \left(\frac{k\Delta l}{2(N_0 + N_d)}\right)^2} - N_c \quad (6)$$

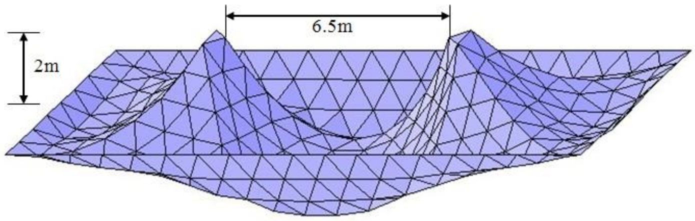

Figure6: Large deformation of compression-free membrane

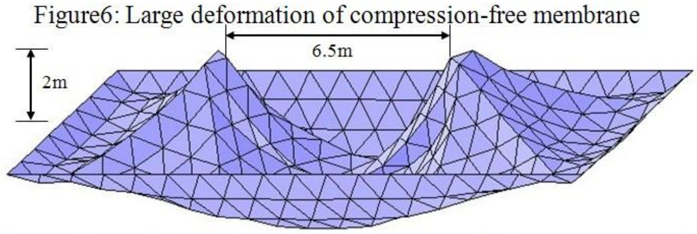

Figure7: Large deformation of membrane considering compressive stiffness

4. COMPUTATIONAL EXAMPLE

Figure6 and 7 show a comparison between the compression-free membrane and membrane considering compressive stiffness. The wrinkle is caused in the figure7 because of considering the small compressive stiffness. The details will be presented in the conference.

Reference:井嶋克志，帯屋洋之，川崎徳明：空間柔ケーブルによる非抗圧膜構造モデルの有限変位解析，構造工学論文集，Vol.55A，pp.11-22,2009.3.