Simultaneous Multi-scale Inversion Method for Estimating Source Location and Crust Structure

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1. Introduction

Earthquake motion is known to be strongly influenced by the crust structure and the source parameters. The crust structure is mainly estimated by geophysical exploration, while the source parameters are estimated with the assumption of a horizontally-layered crust structure model. The estimation accuracy of crust structure can be improved by seismic waveform inversion. However, the estimation accuracy is insufficient if we use the source parameters derived using the said assumption. In this paper, a consideration of simultaneous estimation of source location and crust structure is carried out towards highly-accurate prediction of strong ground motion. A Finite Element Method (FEM) which can automatically generate the model of complicated crust structure with less computational resources[1)] (MCFEM) and an inversion method which repeats Finite Element (FE) analysis from low to high resolution gradually are combined and applied as multi-scale inversion modeling method.

2. Methodology

Source parameter inversion is conducted by solving optimization problem of minimizing residual of reference and synthetic response in unknown source parameters and known crust structure. Similarly, the crust structure inversion is carried out by solving optimization problem of minimizing residual of reference and synthetic response in unknown crust structure and known source parameters. The following objective function is used in the inversion problem.

$$f(\mathbf{x}) = \sum_{r=1}^{NR}\sum_{i=1}^{3}\sum_{t=1}^{T} |(\tilde{u}_i(\mathbf{s}_r, t) - u_i(\mathbf{s}_r, t; \mathbf{x})|. \qquad (1)$$

Therein, $\mathbf{x}$, $\mathbf{s}$, NR, r, i, T, t are: model parameter, receiver points, total number of receivers, receiver counter, spatial component, time duration, and time variable, respectively. $\tilde{u}$ is the displacement vector of the reference waveform, and u is the displacement vector of synthetic waveforms. In this paper, the quasi-Newton method, Broyden-Fletcher-Goldfarb-Shanno (BFGS) method[2)], is used as optimization method to minimize Eq.(1). MCFEM[1)] is used as forward modeling method.

The purpose of this research is to improve the accuracy of a proposed low-resolution model (∼0.2 Hz) to high-resolution model (∼1.0 Hz). However, it is difficult to obtain the global minimum directly for a high-resolution model using a low-resolution model as the initial solution. The convergence to local minimum is prevented by repetitive computations from low to high-resolution gradually (multi-scale). 3D FE analysis of a low-resolution model does not require much computational load as compared to a high-resolution model. Thus, the computation speed is improved, since inverse problem requires many repeated computations.

3. Simultaneous Inversion of Source location and Crust Structure

We conduct simultaneous estimation of the unknown geometry of the boundary interface of a two-layered crust model and the unknown source location. The target model is shown in **Figure1**a). The geometry of boundary interface between upper layer and lower layer is shown in **Figure1**b), and settings of synthetic model and observation points are shown in **Table1** and **Figure1**c), respectively. The boundary interface with 4 parameters, as shown in **Figure1**b, d), is constructed using the following procedure: First, the elevation of nodes on the edge of boundary interface are fixed to -1500m, and the 4 nodes (a, b, c, d) within the boundary interface are set as arbitrary crust parameters. Then, the elevation of nodes except these 4 nodes (a, b, c, d) and on the edge of boundary interface are computed by interpolation using minimum curvature method[3)]. The estimation of crust structure is carried out by optimizing these 4 parameters. Inversion result is solved by optimization of minimizing Eq.(1) as with increasing target frequencies of 0.25 Hz, 0.5 Hz, and 1.0 Hz. The source location and crust parameters are shown in **Table2**. **Figure2** shows geometry of interface of the reference solution and the inversion results. The x-component velocity waveforms at 4 observation points are shown in **Figure3**. These waveforms are bandpass-filtered (0.0-0.1, 0.9-1.0 Hz), the target resolution. Behavior of the objective function is shown in **Figure4**. Although examination of improving the accuracy is needed, these results show the convergence to reference solution for both the value of parameters and velocity waveforms. Therefore, it is verified that this inversion method can estimate source location and crust structure simultaneously.

Key Words: *Simultaneous multi-scale inversion, Source location inversion, Crust structure inversion*
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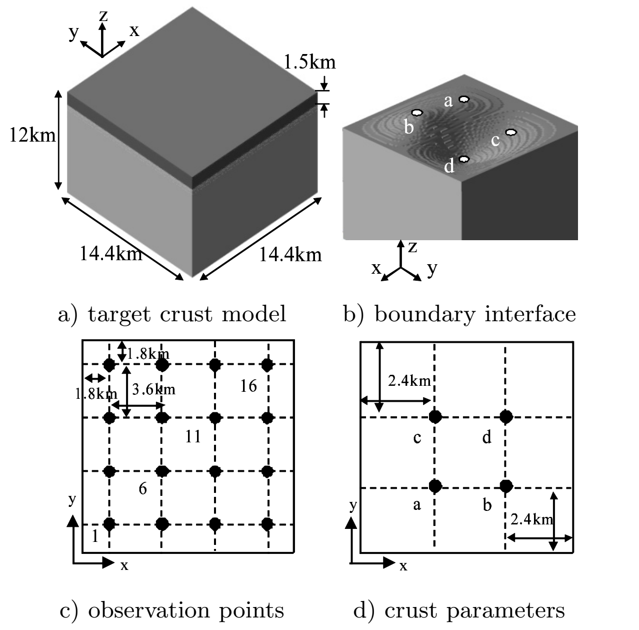
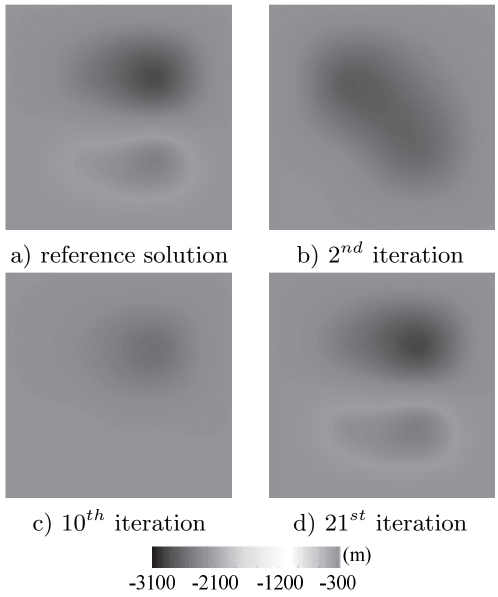


a) target crust model b) boundary interface

c) observation points d) crust parameters

Figure1 The 3D inversion problem settings.



a) reference solution b) 2^{nd} iteration

c) 10^{th} iteration d) 21^{st} iteration

Figure2 Inversion interface results.

Table1 Problem settings.

1^{st}layer:	ρ = 1900 kg/m^3
	V$_p$ = 2300 m/sec
	V$_s$ = 1500 m/sec
	Q$_p$ = 100
	Q$_s$ = 100
2^{nd}layer:	ρ = 2500 kg/m^3
	V$_p$ = 4500 m/sec
	V$_s$ = 3000 m/sec
	Q$_p$ = 300
	Q$_s$ = 300
strike, dip, rake:	30°, 40°, 50°
seismic moment (M$_0$):	1.0×10^{17} Nm
rise time (T$_0$):	2.0 sec
excitation:	$\mathrm{M_0}\ (2\ \mathrm{t}^2/\mathrm{T}_0^2)\quad 0 \le \mathrm{t} \le \frac{\mathrm{T_0}}{2}$
	$\mathrm{M_0}\ (1\text{-}2\ (\mathrm{t}\text{-}\mathrm{T_0})^2/\mathrm{T}_0^2)\quad \frac{\mathrm{T_0}}{2} \le \mathrm{t} \le \mathrm{T_0}$
	$\mathrm{T_0}\quad \mathrm{t} \le \mathrm{M_0}$

Table2 Crust (*) and source location (**) parameters.

	Initial solution	Inversion result	Reference solution
a* (m)	-1500	-893	-1000
b* (m)	-1500	-565	-600
c* (m)	-1500	-2119	-2100
d* (m)	-1500	-3024	-3000
x** (m)	750	150	150
y** (m)	-450	150	150
z** (m)	-7000	-5799	-5850

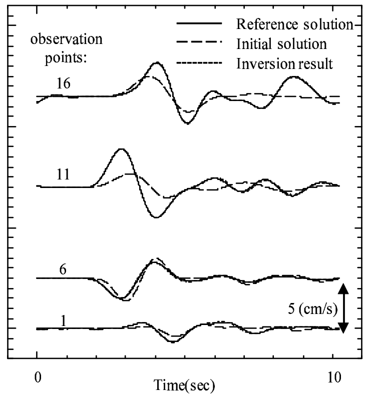


Figure3 X-component velocity waveforms at 4 observation points (bandpass-filtered 0.0-0.1, 0.9-1.0 Hz).

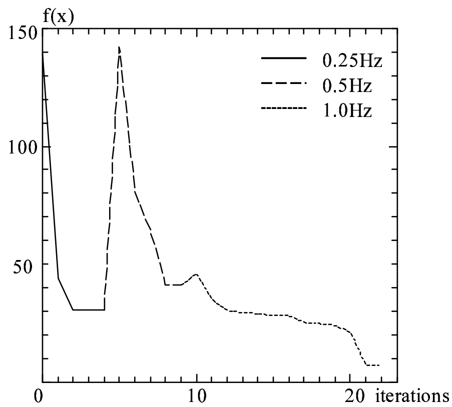


Figure4 Behavior of the objective function.

4. Conclusion

In this paper, simultaneous inversion of source location and crust structure by using multi-scale method is proposed . In the future, we aim to estimate actual source parameters and crust structure by applying the inversion method to past observed earthquakes.

References

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