Structural Identification of Beams by Use of Additional Known Masses and its Application to a Real-life Bridge

University of Tokyo Student Member ○ Hung Minh DINH
University of Tokyo Member Tomonori NAGAYAMA
University of Tokyo Fellow Yozo FUJINO

1. Introduction

Maintaining civil structures has become extremely important. To assess structures' condition, the authors had previously proposed a structural parameter identification method for discrete damped structures regardless of baseline vibration measurement(1). Modal identification of the system is performed under various mass perturbation conditions, which are created by adding known masses(2). Structural parameters consistent with the identified modal parameters are determined. In this study, at first the extension of the proposed method to a beam structure is numerically validated. Finally, the proposed method is applied to structural identification of a real-life bridge.

2. Algorithm

The governing differential equation of a linear time invariant dynamic system is as follows

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{f}(t) \qquad (1)$$

where **M**, **C**, **K** are mass, damping, stiffness matrices, and **x**(t), **f**(t) are displacement and external force vectors respectively. After the state-space transformation, the system's eigenvalue problem becomes

$$\mathbf{BX}=-\mathbf{AX}\Lambda \qquad (2)$$

where $\mathbf{A}=\begin{bmatrix}\mathbf{C} & \mathbf{M}\\ \mathbf{M} & \mathbf{0}\end{bmatrix}$, $\mathbf{B}=\begin{bmatrix}\mathbf{K} & \mathbf{0}\\ \mathbf{0} & -\mathbf{M}\end{bmatrix}$

and Λ , $\mathbf{X}$ are the eigenvalue and eigenvector matrices respectively. When masses are added to the structure, its mass matrix is changed by $\mathbf{M}_a$, and Eq. (2) becomes

$$(\mathbf{B}+\mathbf{B}_a)\mathbf{X}_a=-(\mathbf{A}+\mathbf{A}_a)\mathbf{X}_a\Lambda_a \qquad (3)$$

where $\mathbf{A}_a=\begin{bmatrix}\mathbf{0} & \mathbf{M}_a\\ \mathbf{M}_a & \mathbf{0}\end{bmatrix}$, $\mathbf{B}_a=\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & -\mathbf{M}_a\end{bmatrix}$

Pre-multiplying both sides of Eq. (3) with $\mathbf{X}^T$ and rearranging it yields

$$\mathbf{X}^T\mathbf{A}\mathbf{X}_a\Lambda_a+\mathbf{X}^T\mathbf{B}\mathbf{X}_a=-(\mathbf{X}^T\mathbf{A}_a\mathbf{X}_a\Lambda_a+\mathbf{X}^T\mathbf{B}_a\mathbf{X}_a) \qquad (4)$$

Matching the (i,j)th element of both sides of Eq. (4) yields

$$\lambda_{aj}\phi_i^T\mathbf{C}\phi_{aj}+\phi_i^T\mathbf{K}\phi_{aj}+\lambda_{aj}^2\phi_i^T\mathbf{M}\phi_{aj}=-\lambda_{aj}^2\phi_i^T\mathbf{M}_a\phi_{aj} \qquad (5)$$

Eq. (5) can be rewritten as an equation of elemental structural parameters m, k, and c as follows.

$$\sum^{all\, m_p}\left\{\lambda_{aj}^2\phi_i^T\frac{\partial\mathbf{M}}{\partial m_p}\phi_{aj}^T\right\}m_p+\sum^{all\, k_q}\left\{\phi_i^T\frac{\partial\mathbf{K}}{\partial k_q}\phi_{aj}^T\right\}k_q+$$
$$+\sum^{all\, c_r}\left\{\lambda_{aj}\phi_i^T\frac{\partial\mathbf{C}}{\partial c_r}\phi_{aj}^T\right\}c_r=-\lambda_{aj}^2\phi_i^T\mathbf{M}_a\phi_{aj}^T \qquad (6)$$

As $\mathbf{M}_a$ is known, and natural frequencies, mode shapes are obtainable from modal identification, Eq. (6) for various (i,j) is solved for structural parameters. $\mathbf{M}_a$ can also be changed in order to obtain various mass perturbation conditions.

3. Numerical validation on a beam structure

The proposed method has been proved capable of structural identification of a discrete spring-mass system(1). For this method to be able to apply to a beam structure, such as the one in Fig. 1, the beam has to be discretized into discrete elements. Subsequently its flexural stiffness EI can be identified using equation Eq. (6). When translational and rotational degrees-of-freedom (DOFs) of all nodes can be obtained, each elemental EI is accurately identified as in Fig. 2, where solid bars and hollow bars represent identified and exact values respectively.

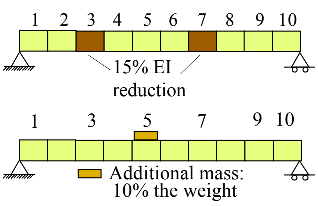


Fig. 1: A reference beam model

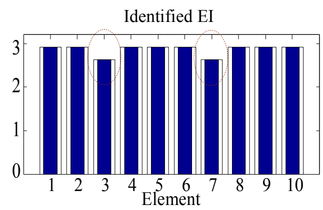
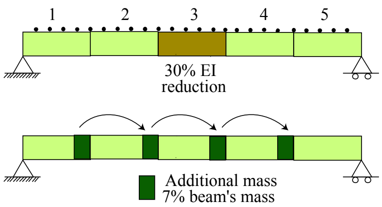


Fig. 2: Stiffness i.d. result

When rotational DOFs are unobservable, which is usual in practice, such as the one in Fig. 3, nodal rotational DOFs are interpolated from densely measured translational DOFs at dotted locations. The identified results of elemental EI as in Fig. 4 show good agreement with the exact values. Thus, the method shows a possibility of application to real structures.

Keyword: structural parameter, modal parameter, identification, numerical validation, experiment, damage detection
Address: Hongo 7-3-1, Bunkyo-ku, Tokyo 113-8656, Japan; Tel: 03-5846-6099, Fax: 03-5841-7454; E-mail: dinh@bridge.t.u-tokyo.ac.jp



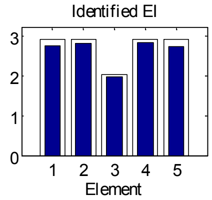


Fig. 3: Beam model w/o rotation data **Fig. 4**: Stiffness i.d. result

4. Application to a real-life bridge

In order to experimentally validate the proposed method to a real-life structure, a test was conducted with a bridge as described in Fig. 5. The bridge consists of 4 steel girders and a concrete slab. The girders are instrumented with 12 accelerometers, marked by solid squares in Fig. 5. The bridge is mass-perturbed 4 times by placing steel blocks whose total weight is up to 15 tons, or about 12% of the bridge's weight, each time at a different location from P1 to P4 as marked by stars in Fig. 5. In each condition, the bridge is excited by dropping a sandbag 3 times in a row onto its slab from the air at 5 different locations marked by circles.

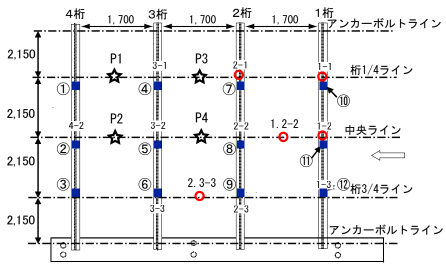


Fig. 5: Bridge schematics and sensors layout

Fig. 6 shows natural frequencies of the first bending mode of all trials. Although there is a small scattering of frequency due to slightly weak repeatability and impact location change, changes of the first natural frequency due to added masses show clear trends, ranging from around 3% when the mass is added at P3, P4, to as high as 7% at P1, P2.

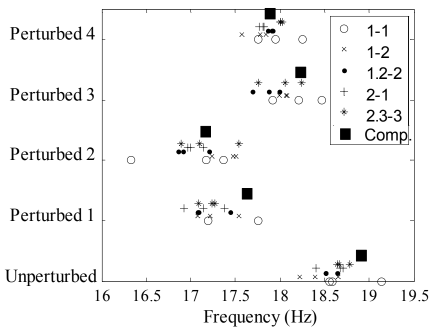
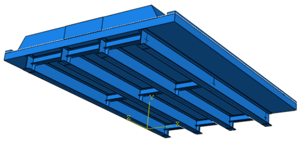


Fig. 6: 1st bending frequencies in tests and model computation

In order to confirm this trend, the bridge is modeled by Abaqus® as shown in Fig. 7, using material inputs from preliminary tests and its geometry from design drawing. The boundary conditions are fixed-roller for the girders and partially fixed-fixed for the concrete slab due to its continuity at both ends. First bending modes of the modeled bridge in all conditions are computed. The first mode shape in one condition is shown in Fig. 8 as an example.



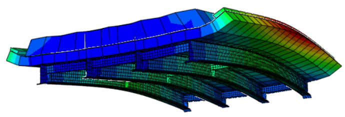


Fig. 7: Bridge analysis model **Fig. 8**: Computed 1st bending mode

All the natural frequencies are plotted as solid squares in Fig. 6, and they show good agreement with the corresponding measurement results. Significant changes of the first frequency of a real-life bridge due to added mass have been confirmed by both tests and model simulation. However, application of the proposed method to identify structural parameters of this bridge has been unsuccessful due to the scarcity of modal properties obtained from data measurement, in which high modes could not be properly identified. This difficulty indicates that the idea of adding known mass to obtain more information on a structure needs to be utilized in a way so that a small number of modal properties are sufficient for identification. For this purpose, model updating method combined with mass perturbation is expected to be a good solution. This will be the direction of the authors' future research in order to tackle the difficulties of structural identification of real-life structures.

5. Conclusions

Extension of the previous research to structural identification of a beam structure has been numerically validated. Although its direct application to a real-life bridge was unsuccessful, the test gave insight into how a real-life bridge's frequency changes due to added mass. Efforts will be made to utilize added mass in model updating in expectation of enhancing structural identification capability.

References

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