Critical Pressure Prediction in Rock Grouting by Using the Fracture Mechanics Principles

Khosravi M.H.[1], Majdi A.[2], Pipatpongsa T.[3], Hideki Ohta[4]

Abstract
As an important concern of grouting experts and engineers, optimal grouting pressures for executing a good practice has been a complicated subject for decades. However, there is no dominating concept of it either in literature or in practice. In this study based on hydraulic fracturing in cracks while the grouting fluid under pressure is injecting to it, using the fracture mechanics principles, a new method for predicting the critical grouting pressures was developed.

1. Introduction

Abe et al., (1979), considered a vertical penny-shaped fracture growth due to a continuously injected fracturing fluid from an inlet borehole to investigate the heat extraction from the fracture in hot dry rock. In Fig. 1 illustrates an inclined penny-shaped fracture with the radius of a, normal vector n and a trend angle α. The fluid is injected from a small borehole with the radius of R at the center of the crack located at a distance h_o below the ground surface.

2. Crack tip stress intensity factor

The crack tip stress intensity factor for a penny-shaped vertical crack can be obtained by using the equation (1) (Abe et. al., 1979).

$$K_I = \frac{\sqrt{2a}}{\pi}\left(P_o - \sigma_n^o + \frac{2}{3}ga(k_o\rho_r - \rho_f)\cos\theta\right) \qquad (1)$$

Considering the independency between P_o, σ_n^o and trend angle of crack surface, the crack tip stress intensity factor for a penny-shaped inclined crack can be obtained by using the following equation:

$$K_I = \frac{\sqrt{2a}}{\pi}\left(P_o - \sigma_n^o + \frac{2}{3}ga(k_o\rho_r - \rho_f)\cos\theta\sin\alpha\right) \qquad (2)$$

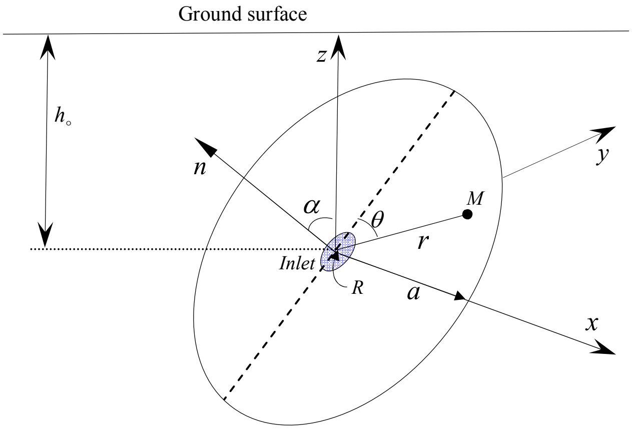

Figure 1: Geometry and coordinate system for inclined penny-shaped crack

Where K_I is the crack tip stress intensity factor at the point of (a,θ,α) on the crack wall, a is the radius of crack, P_o is the fluid pressure in the h_o depth of the borehole, σ_n^o is the normal stress at the center of crack, g is the gravity acceleration, k_o is the coefficient of in-situ rock pressure, ρ_r is the density of rock, ρ_f is the density of fluid injecting to borehole, θ is the angle between M direction and vertical on the crack surface and α is the trend angle of crack surface.

The normal stress at the center of crack by considering the coefficient of in-situ rock pressure ($k_o = \frac{\sigma_y}{\sigma_z}$) can be calculated as;

$$\sigma_n^o = \sigma_z(\cos\alpha + k_o\sin\alpha) \qquad (3)$$

Where σ_z is the vertical in situ stress and σ_y is the horizontal in situ stress perpendicular to crack alignment.

It can be observed from equation (2) that K_I varies along the fracture front, i.e. depends on θ. Since $k_o\rho_r$ is usually larger than ρ_f, thus K_I is maximized at the top edge of crack, i.e. $\theta = 0^o$, and minimized at the bottom of the crack, i.e. $\theta = 180^o$. Therefore the crack tends to initiate from the top and propagate upward and in the case of hydraulic fracturing due to extra pressure of fluid, it will happen at the top edge of crack. Substituting the $\theta = 0^o$ into equation (2), yields for the stress intensity factor at the top edge of crack:

$$K_I = \frac{\sqrt{2a}}{\pi}\left(P_o - \sigma_n^o + \frac{2}{3}ga(k_o\rho_r - \rho_f)\sin\alpha\right) \qquad (4)$$

1. Research Student, Tokyo Institute of Technology, Tokyo, Japan.
2. Associate Professor, University of Tehran, Tehran, Iran.
3. Associate Professor, Tokyo Institute of Technology, Tokyo, Japan.
4. Professor, Tokyo Institute of Technology, Tokyo, Japan.

3. Pressure prediction

Unstable crack propagation occurs when fluid pressure reaches the critical amount ($P_o = P_b$) which leads to stress intensity factor reaching the fracture toughness of the rock mass ($K_I = K_{IC}$) (Abe et. al., 1979). In this case equation (4) can be expressed as follows:

$$K_{IC} = \frac{\sqrt{2a}}{\pi}\left(P_b - \sigma_n^o + \frac{2}{3} ga(k_o\rho_r - \rho_f)\sin\alpha \right) \tag{5}$$

By solving the equation (5) for P_b and substituting equation (3) into it, the critical fluid pressure yields the following:

$$P_b = \frac{\pi K_{IC}}{\sqrt{2a}} - \frac{2}{3} ga(k_o\rho_r - \rho_f)\sin\alpha + \rho_r g h_o(\cos\alpha + k_o \sin\alpha) \tag{6}$$

In a certain grouting practice, the critical fluid pressure (p_b) in any section of borehole depends on the fracture toughness of the rock mass (K_{IC}), rock mass density (ρ_r), crack length (a) and the trend angle of crack plane (α). Therefore, in order to determine the critical grouting pressure in a foundation, using the equation (6) for any of its constituent rocks, p_b can be determined. Then for any of them the grouting pressures can be shown versus section depth in a diagram ($p_b - h_o$). Finally, the graph which suggests the minimum pressures will be determined as critical grouting graph. In fact the left hand of this graph is safe for grouting. But if the grouting pressure exceeds to the right hand, the hydraulic fracturing probability is very high.

4. Critical grouting pressures for sandstone

In this section by assuming the vertical penny-shaped cracks ($\alpha = 0^o$), the critical grouting pressures for different crack length in sandstone are determined. In Fig. 2, by assuming $K_{IC} = 0.67 MPa\sqrt{m}$, $\rho_r = 2400 kg/m^3$, $g = 9.81 m/s^2$, $k_o = 1.2$ and $\rho_f = 1600 \frac{kg}{m^3}$, the critical fluid pressure versus crack length for sandstone is illustrated. In order to compare fracture mechanics approach with empirical approaches, the empirical graphs developed by “American School” and “European School” (Ewert, 1997) are also shown. It can be observed from this figure that if there are tiny cracks in rock mass, it is possible to apply high pressures for grouting, even in shallow sections. Increasing of the crack lengths leads to decreasing of the rock mass strength; hence in longer cracks the hydraulic fracturing will occur in lower pressures.

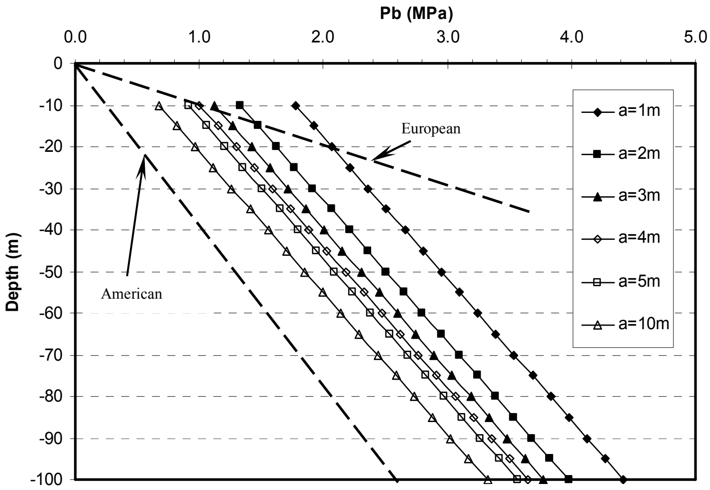

Figure 2: The critical fluid pressure graphs in corresponding to crack length for sandstone

It is also obvious that “European School” graph suggests very higher pressures compared with fracture mechanic approach in which the probability of hydraulic fracturing is very high. On the other hand the pressures suggested by “American School” graph are lower than those determined by fracture mechanic approach.

5. Conclusions

In order to predict the critical grouting pressures in rock mass consists of different rocks; the critical grouting pressures graphs for any of the constituent rocks have to be drawn. Then the index graph is the one that suggests the lower pressures. The left zone of this graph is considered as the safe area for grouting but if the pressure in a grouting section increases to the right zone of it, the hydraulic fracturing process will happen.

Comparing the fracture mechanics approach with empirical graphs suggested by “European school” and “American school”, shows that the suggested pressures by “American school” graph are very low and is good for poor rock masses with lots of cracks. But for grouting a sound rock mass with tiny cracks, the “European school” suggests more suitable pressures.

References

Abe, H., Keer, L. M. and Mura, T., 1979, Theoretical study of hydraulic fractured penny-shaped cracks in hot, dry rocks, Int. J. Num. Ana. Methods in Geomech., 3, 79-96.

Ewert, F.K., 1997, "Permeability, groutability and grouting of rocks related to dam sites; Part 4: groutability and grouting of rock”, Dam Engineering, vol. 8, issue 4, pp. 271-325.