Ambient vibration based automatic estimation of bridge dynamic characteristics by Balanced Stochastic Realization(BSR) theory

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1. Introduction This study explains about the method for the estimation of bridge dynamic characteristics by Balanced Stochastic Realization(BSR) theory based on ambient vibration. Covariance matrix were derived from past and future ambient vibration data block matrix to estimate the coefficient matrices for state space model of bridge. Proposed method were applied to the existing Kabashima bridge and its dynamic characteristics were estimated by the multipoint ambient vibration measurements for three different conditions such as strong wind, weak wind, and moving vehicle condition.

2. Modeling of bridge The dynamic behavior of bridge is expressed by $ndofs$ equation of motion and its discretized state space model will be as followings

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{f}(k) \tag{1}$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{f}(k) \tag{2}$$

where, $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{f} \in \mathbf{R}^m$, $\mathbf{y} \in \mathbf{R}^r$ are the variables and $\mathbf{A} \in \mathbf{R}^{n\times n}, \mathbf{B} \in \mathbf{R}^{n\times m}$, $\mathbf{C} \in \mathbf{R}^{r\times n}$ are the coefficient matrices.

3. Balanced Stochastic Realization(BSR-I) theory Let $\{\mathbf{y(t)}, t = 0, 1, 2, \cdots N + 2k + -2\}$, be the measured data in finite time .The block Toeplitz matrix($\mathbf{T}^+$, $\mathbf{T}_-$) and Hankel matrix ($\mathbf{H(0)}$) are obtained from past and future response data block $\mathbf{Y}_p \in \mathbf{R}^{mk\times N}$ and $\mathbf{Y}_f \in \mathbf{R}^{mk\times N}$ respectively. Covariance of measerement data were obtained by the LQ orthogonal decomposition of data block matrix.

$$\begin{bmatrix} \mathbf{\Sigma}_{pp} & \mathbf{\Sigma}_{pf} \\ \mathbf{\Sigma}_{fp} & \mathbf{\Sigma}_{ff} \end{bmatrix} = \frac{1}{N} \begin{bmatrix} \mathbf{Y}_p \\ \mathbf{Y}_f \end{bmatrix} \begin{bmatrix} \mathbf{Y}_p^T & \mathbf{Y}_f^T \end{bmatrix} \quad \text{or,} \quad \begin{bmatrix} \mathbf{\Sigma}_{pp} & \mathbf{\Sigma}_{pf} \\ \mathbf{\Sigma}_{fp} & \mathbf{\Sigma}_{ff} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{11} & \mathbf{0} \\ \mathbf{L}_{21} & \mathbf{L}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{L}_{11}^T & \mathbf{L}_{21}^T \\ \mathbf{0} & \mathbf{L}_{22}^T \end{bmatrix} \tag{3}$$

in which $\mathbf{\Sigma}_{pf}$, $\mathbf{\Sigma}_{ff}$, $\mathbf{\Sigma}_{pp}$ are finite dimensional approximations over the infinite matrices $\mathbf{H(0)}$, $\mathbf{T}^+$, $\mathbf{T}_-$ respectively.

Square root matrices of the covariance matrices Σ_{ff}, Σ_{pp} such that

$$\mathbf{\Sigma}_{pp} = \mathbf{L}\mathbf{L}^T \quad , \quad \mathbf{\Sigma}_{ff} = \mathbf{M}\mathbf{M}^T \tag{4}$$

Executing Singular Value Decomposition (SVD) of the normalized covariance matrix Σ_{pf} as

$$\mathbf{L}^{-1}\mathbf{\Sigma}_{pf}\mathbf{M}^{-T} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \cong \mathbf{U}_s\mathbf{\Sigma}_s\mathbf{V}_s^T \tag{5}$$

where, $\mathbf{\Sigma}_s$ is obtained by deleting too small singular values of $\mathbf{\Sigma}$.

The extended observability and reachability matrices are

$$\mathbf{P}_k = \mathbf{L}\mathbf{U}_s\mathbf{\Sigma}_s^{\frac{1}{2}} \qquad \mathbf{Q}_k = \mathbf{\Sigma}_s^{\frac{1}{2}}\mathbf{V}_s\mathbf{M}^T \tag{6}$$

The coefficient matrices $\mathbf{A}$, $\mathbf{C}$, and $\mathbf{B}$ can be computed by the following equation.

$$\mathbf{A} = \underline{\mathbf{P}}_k^+\bar{\mathbf{P}}_k \qquad \mathbf{C} = \mathbf{P}_k(1{:}p, :) \quad \mathbf{B}^T = \mathbf{Q}_k(:, 1:p) \tag{7}$$

where, $\underline{\mathbf{P}}_k = \mathbf{P}_k(1:(k-1)p, :)$ and $\bar{\mathbf{P}}_k = \mathbf{P}_k(p+1{:}kp, :)$are the matrices formed by excluding the lower and upper p rows of $\mathbf{P}_k$.

4. Balanced Stochastic Realization (BSR-II) theory Approximation of state vector can be computed by using the eq.(4) as

$$\bar{\mathbf{X}}_k = \hat{\mathbf{\Sigma}}_s^{\frac{1}{2}}\hat{\mathbf{V}}_s^T\mathbf{M}^{-1} \quad , \quad \check{\mathbf{Y}}_{0|k-1} \in \mathbf{R}^{n\times N} \tag{8}$$

The coefficient matrices $\mathbf{A}$ and $\mathbf{C}$ are obtained by the following relation.

$$\begin{bmatrix} \hat{\mathbf{X}}_{k+1} \\ \hat{\mathbf{Y}}_{k|k} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{C} \end{bmatrix} \hat{\mathbf{X}}_k + \begin{bmatrix} \rho_w \\ \rho_v \end{bmatrix} \tag{9}$$

Thus we can estimate the coefficient matrix $\mathbf{A}$ and $\mathbf{C}$ by BSR-II method .

Fig.1 Kabashima bridge

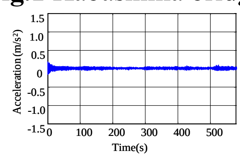

Fig.2 Weak wind data

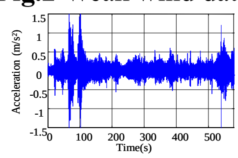

Fig.3 Strong wind data

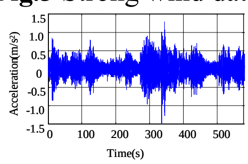

Fig.4 Passing vehicle data

5. Dynamic characteristics estimation The complex conjugate eigenvalues $\lambda_k = X_{\mathrm{Re}}^k \pm X_{\mathrm{Im}}^k$ and eigenvectors are obtained by the eigenvalue analysis of system matrix $\mathbf{A}$.Frequencies and damping ratios of the system(bridge) will be obtained from the real and imaginary part of eigenvalue by the following equations.

$$h_k\omega_k = -\log\sqrt{{X_{\mathrm{Re}}^k}^2 + {X_{\mathrm{Im}}^k}^2}/\Delta \qquad \text{and} \qquad \omega_k\sqrt{1-h_k^2} = \tan^{-1}\frac{X_{\mathrm{Im}}^k}{X_{\mathrm{Re}}^k}/\Delta \tag{10}$$

The vibration mode can be found from the eigenvectors.

Keyword: System identification, Ambient vibration, Realization theory, Dynamic characteristics.

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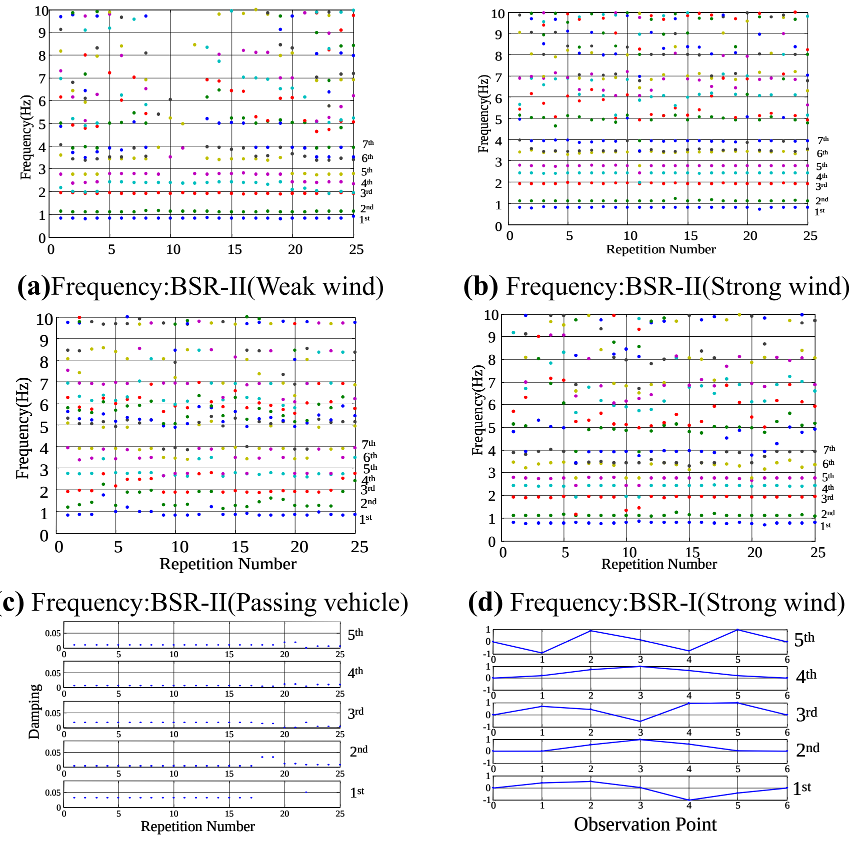
6.Application to ambient vibration measurement The proposed method were applied to Kabashima bridge (**Fig.1**) which is situated at Nagasaki prefecture. For executing the multipoint ambient vibration measurement, five accelerometers were placed at various locations along the bridge length. The experiments were conducted considering three different ambient conditions such as weak wind, strong wind, and passing vehicle. The recorded ambient vibration can be shown in **Fig.2, 3,** and **4** for weak wind, strong wind and passing vehicle data respectively, to observed their characteristics. **Fig.2** reveals that measured data possess stationary characteristics with low amplitude; **Fig.3** represents the stationary with high amplitude characteristics whereas **Fig.4** shows the non stationary characteristics.



(a) Frequency:BSR-II(Weak wind) **(b)** Frequency:BSR-II(Strong wind)

(c) Frequency:BSR-II(Passing vehicle) **(d)** Frequency:BSR-I(Strong wind)

(e) Damping: BSR-II(Strong wind) **(f)** Mode:BSR-II(Strong wind)

Fig.5 Estimated dynamic characteristics

7.Considerations on estimation accuracy **Figures 5(a), 5(b),** and **5(c)** are the graphical representation of estimated frequency by BSR-II method for weak wind, strong wind and passing vehicle respectively. **Fig 5(a)** shows the steady estimation upto 5^{th}mode, **Fig.5(b)** shows the steady estimation up to 7^{th} mode frequency whereas **Fig.5(c)** shows the unsteady estimation. Estimated frequency by BSR-II method (refers to **Fig.5(c)**) were more scattered than estimated frequency by BSR-I(refers to **Fig.5(b)**) method. **Fig.5(d)** is the estimated damping by BSR-II method for strong wind data which indicates that the estimated damping also well arranged except some lacks in first mode. The estimated vibration mode by BSR-II method for strong wind can be seen in **Fig.5(e)** .The estimated vibration mode shape remains unchanced for all the cases as they are estimated from average values.The mean values of frequency and damping are shown in **Table 1** for strong wind data by BSR-I and BSR-II method.The coefficient of variation of frequency for BSR-II method is less than 0.5% except 7^{th} mode whereas it is within 8% for BSR-I method. From the damping value we can see that the coefficient of variation of estimated damping for BSR-II method is within 72% except 2^{nd} mode which is lower than BSR-I method.

Table 1 Estimation accuracy(strong wind)

Mode Order		Frequency(Hz)			Damping		
		Mean	Std.	C.V.	Mean	Std.	C.V.
1st	BSR-I	0.774	0.0295	3.81	0.0315	0.0390	126.63
	BSR_II	0.803	0.0020	0.25	0.0549	0.0399	72.75
2nd	BSR-I	1.122	0.0347	3.09	0.0327	0.0589	180.24
	BSR-II	1.108	0.0010	0.00	0.0061	0.0085	139.79
3rd	BSR-I	1.831	0.2273	12.41	0.0281	0.0675	241.61
	BSR-II	1.935	0.0097	0.51	0.0126	0.0059	46.78
4th	BSR-I	2.343	0.1718	7.33	0.0047	0.0066	140.79
	BSR-II	2.416	0.0011	0.05	0.0050	0.0020	40.29
5th	BSR-I	2.701	0.1226	4.53	0.0100	0.0259	258.68
	BSR-II	2.773	0.0083	0.29	0.0082	0.0038	46.33
6th	BSR-I	3.242	0.2610	8.04	0.0132	0.0360	271.59
	BSR-II	3.423	0.0167	0.49	0.0325	0.0178	54.81
7th	BSR-I	3.678	0.2492	6.77	0.0023	0.0195	852.87
	BSR-II	3.884	0.1575	4.01	0.0312	0.0112	35.89

8. Conclusions The proposed method were applied to existing Kabashima bridge and its dynamic characteristics were successfully estimated from ambient vibration by Balanced Stochastic Realization(BSR) method. Based on the experimental result, BSR-II method shows better estimation accuracy over BSR-I method. The estimation accuracy was better under stationary state and high amplitude of ambient vibration and this kind of ambient condition is recommended for ambient vibration experiment.

References : [1] AKAIKE, H.:Markovian representation of stochastic process by canonical variables *SIAM J. Control*, vol. 13, no.1, pp 162-173,1975.

[2] KATAYAMA, T.:*Subspace Methods for System identification*,Springer, 2005.