Structural Parameter Identification by Use of Additional Known Masses and its Application to Damage Detection

University of Tokyo Student Member ○ Hung Minh DINH
University of Tokyo Member Tomonori NAGAYAMA
University of Tokyo Fellow Yozo FUJINO

1. Introduction

Structures in Japan are deteriorating. Considering the national resources invested in those structures and the social impact their failures may inflict, maintaining these structures has become extremely important. In this regard, vibration measurement is expected to allow condition assessment of structures. However, performance evaluation methods utilizing a baseline for comparison require vibration measurement of a structure in its intact condition. However, such baselines are not oftentimes available. In an attempt to tackle this issue, this study proposes a structural parameter identification method for damped structures regardless of baseline vibration measurement. Modal identification of the system is performed under various mass perturbation conditions, which are created by adding known masses(1). Structural parameters consistent with the identified modal parameters are determined. At first, the identification algorithm is developed and then validated through both numerical simulation and experiments on scale-models.

2. Algorithm

The governing differential equation of a linear time invariant dynamic system is as follows

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{f}(t) \quad (1)$$

where **M**, **C**, **K** are mass, damping, stiffness matrices, and **x**(t), **f**(t) are displacement and external force vectors respectively. After the state-space transformation, the system's eigenvalue problem has a similar form to that of Cha's formulation(1)

$$\mathbf{B}\mathbf{X}=-\mathbf{A}\mathbf{X}\Lambda \quad (2)$$

where $\mathbf{A}=\begin{bmatrix}\mathbf{C} & \mathbf{M}\\ \mathbf{M} & \mathbf{0}\end{bmatrix}$, $\mathbf{B}=\begin{bmatrix}\mathbf{K} & \mathbf{0}\\ \mathbf{0} & -\mathbf{M}\end{bmatrix}$

and Λ , $\mathbf{X}$ are the eigenvalue and eigenvector matrices respectively. When masses are added to the structure, its mass matrix is changed by $\mathbf{M}_a$, and Eq. (2) becomes

$$(\mathbf{B}+\mathbf{B}_a)\mathbf{X}_a=-(\mathbf{A}+\mathbf{A}_a)\mathbf{X}_a\Lambda_a \quad (3)$$

where $\mathbf{A}_a=\begin{bmatrix}\mathbf{0} & \mathbf{M}_a\\ \mathbf{M}_a & \mathbf{0}\end{bmatrix}$, $\mathbf{B}_a=\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & -\mathbf{M}_a\end{bmatrix}$

Pre-multiplying both sides of Eq. (3) with $\mathbf{X}^T$ and rearranging it yields

$$\mathbf{X}^T\mathbf{A}\mathbf{X}_a\Lambda_a+\mathbf{X}^T\mathbf{B}\mathbf{X}_a=-(\mathbf{X}^T\mathbf{A}_a\mathbf{X}_a\Lambda_a+\mathbf{X}^T\mathbf{B}_a\mathbf{X}_a) \quad (4)$$

Matching the (i,j)th element of both sides of Eq. (4) yields

$$\lambda_{aj}\phi_i^T\mathbf{C}\phi_{aj}+\phi_i^T\mathbf{K}\phi_{aj}+\lambda_{aj}^2\phi_i^T\mathbf{M}\phi_{aj}=-\lambda_{aj}^2\phi_i^T\mathbf{M}_a\phi_{aj} \quad (5)$$

By utilizing the following relation(2)

$$\mathbf{M}=\sum_{p=1}^{all\,m_p}\frac{\partial\mathbf{M}}{\partial m_p}m_p,\ \mathbf{K}=\sum_{q=1}^{all\,k_q}\frac{\partial\mathbf{K}}{\partial k_q}k_q,\ \mathbf{C}=\sum_{r=1}^{all\,c_r}\frac{\partial\mathbf{C}}{\partial c_r}c_r$$

Eq. (5) can be rewritten as an equation of elemental structural parameters m, k, and c as follows.

$$\sum^{all\,m_p}\left\{\lambda_{aj}^2\phi_i^T\frac{\partial\mathbf{M}}{\partial m_p}\phi_{aj}^T\right\}m_p+\sum^{all\,k_q}\left\{\phi_i^T\frac{\partial\mathbf{K}}{\partial k_q}\phi_{aj}^T\right\}k_q+$$
$$+\sum^{all\,c_r}\left\{\lambda_{aj}\phi_i^T\frac{\partial\mathbf{C}}{\partial c_r}\phi_{aj}^T\right\}c_r=-\lambda_{aj}^2\phi_i^T\mathbf{M}_a\phi_{aj}^T \quad (6)$$

Considering $\mathbf{M}_a$ is known, and natural frequencies, mode shapes are obtainable from modal identification, Eq. (6) for various (i,j) is solved for structural parameters. Note that unlike Cha's method, damping coefficients can be identified. $\mathbf{M}_a$ can also be changed in order to obtain various mass perturbation conditions.

3. Numerical validation

The proposed algorithm is at first investigated in a reference case without observation noise. A four degree-of-freedom shear model as illustrated in Fig. 1 is employed. The model is excited by white noise excitation, and the velocity of each floor is captured. Using the algorithm, mass, stiffness and damping coefficient are identified exactly as shown in Fig. 2, where yellow and blue bars represent the actual and identified values respectively.

Keyword: structural parameter, modal parameter, identification, numerical validation, experimental validation

Address: Hongo 7-3-1, Bunkyo-ku, Tokyo 113-8656, Japan; Tel: 03-5846-6099, Fax: 03-5841-7454; E-mail: dinh@bridge.t.u-tokyo.ac.jp

Because stiffness change is considered the most effective damage indicator, only stiffness identification results are shown from this point.

When 50% white Gaussian observation noise is introduced, modal identification becomes inaccurate, thus results in inaccuracy of structural parameter identification as observed in Fig. 3-a. However, by moving the additional mass to another floor to increase the number of mass perturbation conditions, structural parameter identification becomes accurate as seen on the right in the same figure. Thus, numerical simulation demonstrates that the algorithm is sensitive to observation noise, however its accuracy can be improved by using various mass perturbation conditions.

With the same model, it can be observed from Fig. 4-a that using different combination of modes gives different results, in which using many modes does not necessarily improve the accuracy. For instance, using all modes from 1 to 4 results in worse identification comparing with using modes 1, 2 and 3. To choose the combination of modes which yields the most accurate results as possible, the study proposes an indicator called "Frequency Difference Indicator", or *FDI*. It is defined as the summed difference between the measured natural frequencies and their corresponding regenerated natural frequencies by using identified structural parameters of all modes in all states of the structure, as in Eq. (7).

$$FDI = \sum^{all\,states} \sum^{all\,modes} \left| \frac{\omega_{ij}^{regenerated} - \omega_{ij}^{measured}}{\omega_{ij}^{measured}} \right| \qquad (7)$$

Fig. 4-a demonstrates that FDI is consistent with stiffness error because small FDI indicates small stiffness error.

4. Experimental validation

A 4-storey building scale-model whose 1st floor columns have weaker stiffness than the rest is instrumented with velocimeters as illustrated in Fig. 5. The model is excited by a hammer, and the velocity profile of all floors are captured. The improvement of stiffness identification accuracy when mass perturbation conditions increase can be observed in Fig. 3-b. In each condition, a mass equivalent to 10% of the floor weight is attached to a different floor. The improvement of identification accuracy by modes selection and its consistency with FDI are also confirmed as shown in Fig. 4-b, where the combination of mode 2 and 3 whose FDI is minimum gives the most accurate identification.

5. Conclusions

In this study, a structural parameter identification algorithm has been developed. Identification accuracy can be improved by using various mass perturbation conditions and selecting a suitable set of modes in calculation. An indicator for mode selection has been proposed. These findings have been validated through both numerical simulation and scale-model experiments.

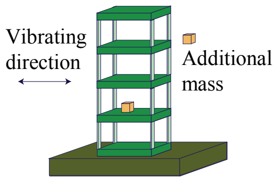

Fig. 1: A four DOF model

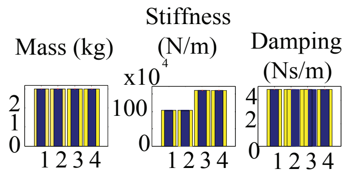

Fig. 2: Reference identification result

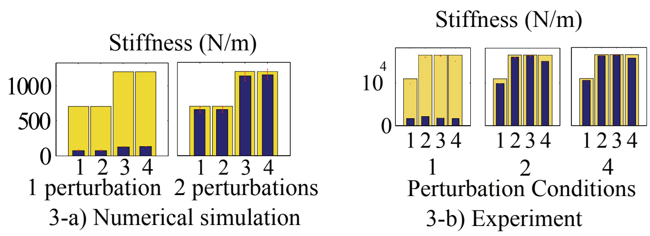

Fig. 3: Mass perturbation conditions on identification accuracy

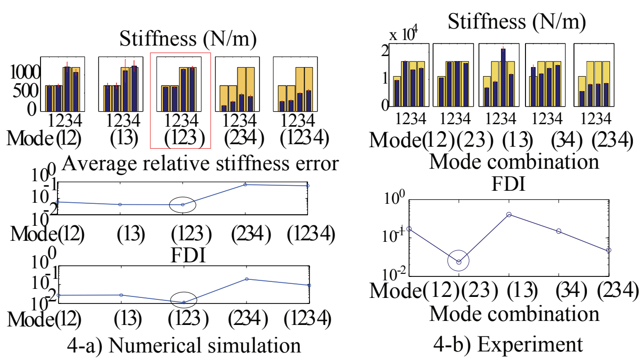

Fig. 4: Validation of mode selection

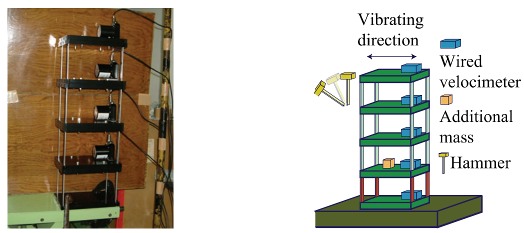

Fig. 5: Actual setup and schematic view of the shear model test

References

[1] AP. D. Cha and L. G. de Pillis. Model Updating by Adding known Masses. *International Journal for Numerical Methods in Engineering*, 50(11):2547-2571, 2001.

[2] A. Abudula and H. Shiojiri. An Efficient Method of Structural Identification. *Transactions of the Japan Society for Computational Engineering and Science*, 7(3):9-14, 2005