Analytical Train-Bridge Interaction Dynamic Simulation to High-speed Railway

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1. Introduction

Speed is an essential part in all characters of transportation. Therefore, all kinds of transportation, including the railway, regard speeding up and shortening travel time as a stepping stone for improving transportation quality and enhancing competition in the transportation market. On October 1, 1964, the commercial operation of the Shinkansen Train in Japan marked the beginning of a new era for high-speed railways.

Shinkansen RC bridges in Japan, constructed over 40 years ago, undertake increasing service loads and severe earthquake loads. Because of the huge amount of kinetic energy carried at high speeds, a train might interact with the bridge and even resonate with it under certain conditions, which will even affect the safety of the passengers. Equally important is the riding comfort of train passengers, which relates closely to the maneuverability of the train during its passage over the bridge at high speed.

In analyzing the *Train-Bridge Interaction* (TBI) systems, two sets of second order equations of motion are written; one is for the trains and the other for the bridge. The interaction forces at the contact points make the two subsystems coupled. This paper formulates TBI by modeling moving trains and a bridge as two systems interacting with each other through the contact forces. By solving the contact forces from the train equations, one can treat them as external forces on the bridge, which can then be solved using conventional finite element procedures. Because of the versatility of such a concept, the train-bridge model can be used in the simulation of various three-dimensional train-bridge systems.

2. Analysis method to decouple the Train-Bridge Interaction problem

The TBI problem is a complicated one in that as the contact points move in time, the system matrices are, in general, time-dependent and must be updated and factorized at each time step in an incremental analysis. Since we want to regard the whole system as two separate parts, a decoupling process is needed in the analysis. Here an uncoupling method by means of the Newmark scheme (finite differences) is used.

1) Train equations

The values of the matrices of the train can be obtained with the Euler-Lagrange equations, and the train model is calculated using Matlab software, modeled as in Figure 1. Using the displacement vector of the 15DOFs of the upper part (car body and bogies), $\{U\}$, and displacement vector of the 12DOFs of the wheels set, $\{W\}$, Equation of motion of the train is expressed as follows:

$$\left[\begin{array}{c|c} M_{uu} & M_{uw} \\ \hline M_{wu} & M_{ww} \end{array}\right]\left\{\begin{array}{c} \ddot{U} \\ \hline \ddot{W} \end{array}\right\}+\left[\begin{array}{c|c} C_{uu} & C_{uw} \\ \hline C_{wu} & C_{ww} \end{array}\right]\left\{\begin{array}{c} \dot{U} \\ \hline \dot{W} \end{array}\right\}+\left[\begin{array}{c|c} K_{uu} & K_{uw} \\ \hline K_{wu} & K_{ww} \end{array}\right]\left\{\begin{array}{c} U \\ \hline W \end{array}\right\}=\left\{\begin{array}{c} 0 \\ \hline F_{we}+R \end{array}\right\} \quad (1)$$

where $\{F_{we}\}$ is the vector of external forces and $\{R\}$ is the unknown vector of the dynamic reactions of the bridge at the positions of the wheels. This can also be written as (2).

$$\begin{aligned} &[M_{uu}]\{\ddot{U}\}+[C_{uu}]\{\dot{U}\}+[K_{uu}]\{U\}=-[M_{uw}]\{\ddot{W}\}-[C_{uw}]\{\dot{W}\}-[K_{uw}]\{W\} \\ &\{R\}=-\{F_{we}\}+[M_{wu}]\{\ddot{U}\}+[C_{wu}]\{\dot{U}\}+[K_{wu}]\{U\}+[M_{ww}]\{\ddot{W}\}+[C_{ww}]\{\dot{W}\}+[K_{ww}]\{W\} \end{aligned} \quad (2)$$

2) Bridge equations

The bridge is modeled and analyzed with the ABAQUS finite element software as in Figure 2. Because of the versatility of the general FEM software, the construction details of any type of the structure can be considered and simulated. Using the vector of the nodal displacements of the bridge, $\{Y\}$, its equation is

$$[M_b]\{\ddot{Y}\}+[C_b]\{\dot{Y}\}+[K_b]\{Y\}=\{F_{be}\}-\{\tilde{R}\} \quad (3)$$

where $\{F_{be}\}$ is the vector of the external forces, and $\tilde{R}=\sum\{N^i\}R^i$ is the vector of the nodal forces corresponding to $\{R\}$; $\{N^i\}$ is the interpolation vector for vertical displacements at the position of wheel *i.* Assuming the wheel does not jump on the rail, the contact condition $\{W\}=\{Y_c\}+\{r_c\}$ must hold; $\{Y_c\}$ is the vector of the bridge deflections and $\{r_c\}$ is that of the rail irregularities at the eight contact points.

3) Decoupling method

The coupled equations are uncoupled by means of the Newmark scheme, of which the order of accuracy is the same as the Newmark finite difference method [1]. Suppose that the displacements, velocities, accelerations of the train and the bridge are known at time t. Defining a time step Δt, the equations of the train at time $t+\Delta t$ are:

$$\begin{aligned} &[M_{uu}]\{\ddot{U}\}_{t+\Delta t}+[C_{uu}]\{\dot{U}\}_{t+\Delta t}+[K_{uu}]\{U\}_{t+\Delta t}=-[C_{uw}]\{\dot{W}\}_{t+\Delta t}-[K_{uw}]\{W\}_{t+\Delta t} \\ &\{R\}=-\{F_{we}\}_{t+\Delta t}+[C_{wu}]\{\dot{U}\}_{t+\Delta t}+[K_{wu}]\{U\}_{t+\Delta t}+[M_{ww}]\{\ddot{W}\}_{t+\Delta t}+[C_{ww}]\{\dot{W}\}_{t+\Delta t}+[K_{ww}]\{W\}_{t+\Delta t} \end{aligned} \quad (4)$$

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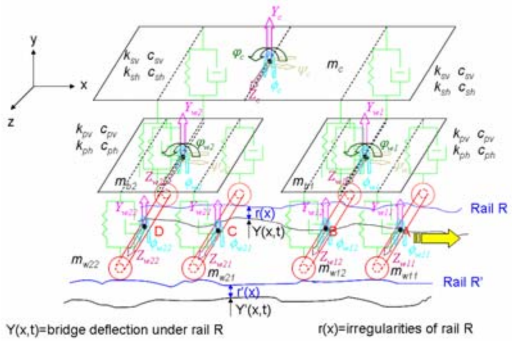

Figure 1. The train model of 27DOFs in Matlab

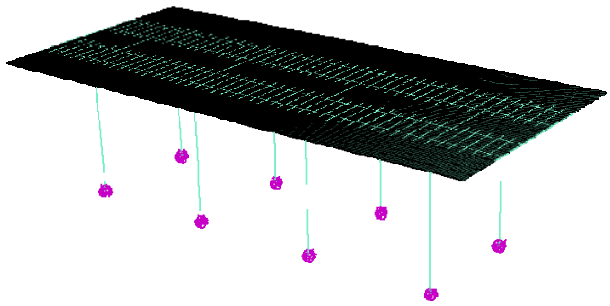
Figure 2. The bridge model in ABAQUS

The Newmark scheme of parameters (β,γ), represented as the following equations,

$$\begin{cases} \{U\}_{t+\Delta t} = \{U\}_t + \{\dot U\}_t \Delta t + \left[\left(\frac{1}{2}-\beta\right)\{\ddot U\}_t + \beta\{\ddot U\}_{t+\Delta t}\right](\Delta t)^2 \\ \{\dot U\}_{t+\Delta t} = \{\dot U\}_t + \left[(1-\gamma)\{\ddot U\}_t + \gamma\{\ddot U\}_{t+\Delta t}\right] \end{cases} \tag{5}$$

expresses $\{\ddot U\}_{t+\Delta t}, \{\dot U\}_{t+\Delta t}, \{U\}_{t+\Delta t}$ as functions of $\{\ddot U\}_t, \{\dot U\}_t, \{U\}_t, \{\ddot W\}_{t+\Delta t}, \{\dot W\}_{t+\Delta t}, \{W\}_{t+\Delta t}$. This relationship is in turn substituted in the second equation. Using the contact condition $\{W\}=\{Y_c\}+\{r_c\}$, the contact force vector is written as

$$\{R\}_{t+\Delta t} = -F_{we,t+\Delta t} + \{q_c\}(\ddot U_t, \dot U_t, U_t) + [M_c]\{\ddot Y_c\}_{t+\Delta t} + [C_c]\{\dot Y_c\}_{t+\Delta t} + [K_c]\{Y_c\}_{t+\Delta t} + [K_c]\{r_c\}_{t+\Delta t} \tag{6}$$

At time $t+\Delta t$, the contact force applied to the bridge is given as

$$\{\tilde R\}_{t+\Delta t} = \underbrace{\sum_i \{N^i\}\left[-F^i_{we,t+\Delta t} + q^i_c(\ddot U_t, \dot U_t, U_t) + \sum_j K^{ij}_c r^j_{c,t+\Delta t}\right]}_{\text{Additional force } \{F*\}} + \underbrace{\left[\sum_{i,j} M^{ij}_c \{N^i\}\{N^j\}^T\right]}_{\text{mass } [M*]}\{\ddot Y\}_{t+\Delta t} + \underbrace{\left[\sum_{i,j} C^{ij}_c \{N^i\}\{N^j\}^T\right]}_{\text{damping} [C*]}\{\dot Y\}_{t+\Delta t} + \underbrace{\left[\sum_{i,j} K^{ij}_c \{N^i\}\{N^j\}^T\right]}_{\text{stiffness } [K*]}\{Y\}_{t+\Delta t} \tag{7}$$

Hence an uncoupled bridge equation is deduced as the following equation.

$$[M_b + M*]\{\ddot Y\}_{t+\Delta t} + [C_b + C*]\{\dot Y\}_{t+\Delta t} + [K_b + K*]\{Y\}_{t+\Delta t} = \{F_{be}\}_{t+\Delta t} - \{F_t *\} \tag{8}$$

$\{Y\}_{t+\Delta t}, \{\dot Y\}_{t+\Delta t}, \{\ddot Y\}_{t+\Delta t}$ are determined from this equation. The contact condition yields $\{W\}_{t+\Delta t}, \{\dot W\}_{t+\Delta t}, \{\ddot W\}_{t+\Delta t}$, which is then used to compute $\{U\}_{t+\Delta t}, \{\dot U\}_{t+\Delta t}, \{\ddot U\}_{t+\Delta t}$. The force $\{F_{t+\Delta t} *\}$ for the next step is computed. Because the additional matrices $M*, K*, C*$ are known from the history of the wheel positions, the equations for the bridge can be solved for the next step.

3. Analytical procedure and analysis result compared with measurement result

According to the analytical method described, the analytical procedure is illustrated as Figure 3. As compared with the field measurement[2], the analysis result agrees well with the measurement result. Figure 4 shows the time-history of the vertical acceleration of a point located on a column of the viaduct. This comparison demonstrates that the analytical method explained in this paper can well simulate the behavior of structures under TBI.

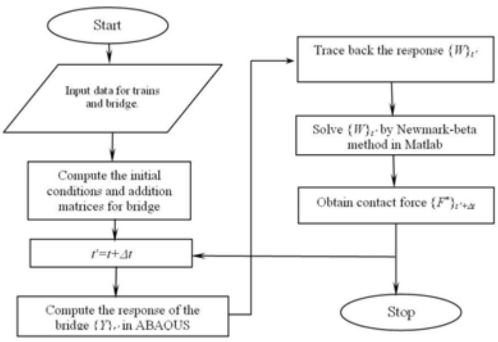

Figure 3. Analytical procedure

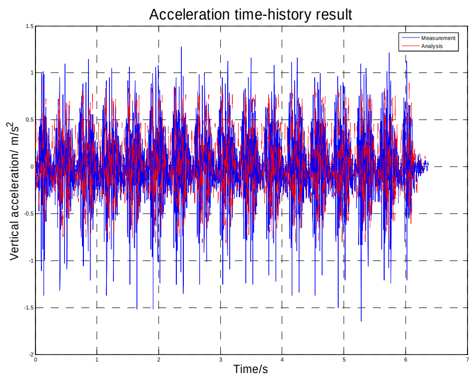

Figure 4. Acceleration results

4. Conclusion and future work

In this paper an effective analysis method of a TBI system is proposed. Because TBI has been taken into account, the model can be used to study not only the bridge vibration response, but also the train response; train response analysis can be utilized as a measure of passengers' riding comfort. The versatility of the method is expected to provide an analysis tool for a variety of applications, such as high-speed train safety analysis under earthquake load or wind load.

Reference

[1] Wu, Y.S., Y.B. Yang, and J.D. Yau, Three-dimensional analysis of train-rail-bridge interaction problems. Vehicle System Dynamics, 2001. 36(1): pp. 1-35.

[2] Jaime Hernandez Jr., Takeshi Miyashita, Hironori Ishii, Phouthaphone Vorabouth, Yozo Fujino. Identification of Modal Characteristics of Shinkansen RC Viaducts using Laser Doppler Vibrometers. In Proceedings of the First Asia-Pacific Workshop on Structural Health Monitoring, Yokohama, Japan, 2006