ANALYSIS AND IDENTIFICATION OF RANDOM PRESSURE FIELDS ON SOME RECTANGULAR CYLINDERS USING COVARIANCE AND SPECTRAL PROPER TRANSFORMATIONS

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1. Introduction

Aerodynamic phenomena of structures in the atmospheric wind flows are generated by spatial distribution and correlation of fluctuating pressure field on structural surface. Understanding and knowledge of the random pressure field and its distribution is possible to interpret mechanisms of excitations, identification and response of aerodynamic phenomena occurring on structure. The Proper Orthogonal Decomposition (POD) was developed by Loeve 1945 and Karhunen 1946, thus also known as the Karhunen-Loeve decomposition. The POD has been branched by either covariance matrix-based or spectral matrix-based proper orthogonal decompositions and associated proper transformations, which depend on how to build up a basic matrix from either zero-time-lag covariance or cross spectral matrices of the multi-variate random processes. Up to now, analyses of the random pressure fields almost have based on the covariance matrix-branched POD due to its straightforward in computation and interpretation (Holmes 1997, Matsumoto et al. 2006). Application to the pressure field analyses based on spectral matrix-branched POD is rare due to its troublesome, but very promising (De Grenet and Ricciardelli 2004).

In this paper, both covariance matrix-based and spectral matrix-based POD of the fluctuating pressure fields on some typical rectangular cylinders are presented for analysis, identification and order-reduced reconstruction of these pressure fields as well as to find out possible relationships between the POD modes and physical phenomena.

2. Proper orthogonal decomposition and its proper transformation branches

In the covariance matrix-branched proper transformation, the random process and its covariance matrix can be reconstructed approximately using truncated number of low-order covariance eigenvalues, eigenvectors (covariance modes):

$$\upsilon(t) = \Theta_\upsilon x_\upsilon(t) \approx \sum_{j=1}^{\widetilde{M}} \theta_{\upsilon j} x_{\upsilon j}(t)\,;\; C_\upsilon = \Theta_\upsilon \Gamma_\upsilon \Theta_\upsilon^T \approx \sum_{j=1}^{\widetilde{M}} \theta_{\upsilon j}\gamma_{\upsilon j}\theta_{\upsilon j}^T \tag{1}$$

where $\widetilde{M}$: number of truncated covariance modes ($\widetilde{M} << N$); $x_\upsilon(t) = \{x_{\upsilon 1}(t), x_{\upsilon 2}(t),\ldots, x_{\upsilon\widetilde{M}}\}^T$: low-order covariance principal coordinates which can be determined from observed data under following expression:

$$x_\upsilon(t) = \Theta_\upsilon^{-1}\upsilon(t) = \upsilon(t)\Theta_\upsilon = \sum_{i=1}^{N} \upsilon_i(t)\theta_{\upsilon i} \tag{2}$$

In the spectral matrix-branched proper transformation, the Fourier transform and the cross spectral density matrix of $\upsilon(t)$ can be represented approximately using truncated number of low-order spectral eigenvalues and eigenvectors (spectral modes):

$$\hat{\upsilon}(n) = \Psi_\upsilon(n)\hat{y}_\upsilon(n) \approx \sum_{j=1}^{\hat{M}} \psi_{\upsilon j}(n)\hat{y}_{\upsilon j}(n)\,;\; S_\upsilon(n) = \Psi_\upsilon(n)\Lambda_\upsilon(n)\Psi_\upsilon^{*T}(n) \approx \sum_{j=1}^{\hat{M}} \psi_{\upsilon j}(n)\lambda_{\upsilon j}(n)\psi_{\upsilon j}^{*T}(n) \tag{3}$$

where $\hat{\upsilon}(n)$: Fourier transform of process $\upsilon(t)$; $\hat{y}_\upsilon(n)$: spectral principal coordinates; $\hat{M}$: number of truncated spectral modes ($\hat{M} << N$); * denotes to complex conjugate operator.

3. Experimental apparatus and measurements

Physical pressure measurements were carried out on three typical rectangular models with slender ratios B/D=1, B/D=1(with Splitter Plate), B/D=5 in the turbulent flows. Artificial turbulent flows were generated by grid device in the wind tunnel at mean wind velocities 3m/s (case1), 6m/s (case 2) and 9m/s (case 3), corresponding to intensities of turbulence I_u=11.46%,I_w=11.23%;I_u=10.54%,I_w=9.28%; I_u=9.52%,I_w=6.65%, respectively. Pressure taps were arranged inside, in chordwise direction and on one surface of models in which model B/D=1 labeled pressure positions from 1 to 10, whereas model B/D=5 from 1 to 19. Unsteady surface pressures were simultaneously measured by the multi-channel pressure measurement system (ZOC23 system). Electric signals were filtered by 100Hz low-pass filters (E3201, NF Design Block Co., Ltd.) before passed through A/D converter (Thinknet DF3422, Pavec Co., Ltd.) with sampling frequency at 1000Hz in 100 seconds.

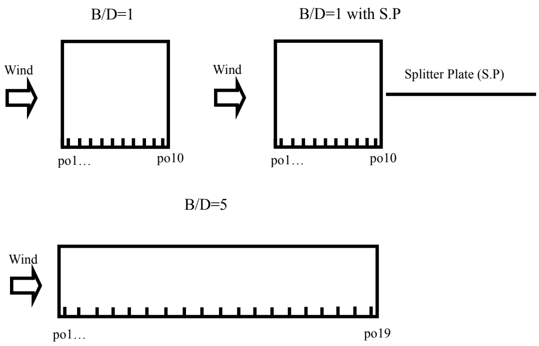

Fig.1 Three experimental models

Bluff body flow around models can be predicted that model B/D=1 is favorable for formation of Karman vortex shedding, where model B/D=5 is typical for formation of separated and reattached flows on model surface. The splitter plate was added to model B/D=1 to suppress effect of Karman vortex.

Keywords: Random pressure field, proper orthogonal decomposition, covariance proper transformation, spectral proper transformation
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4. Results and discussions

Firstly, covariance eigenvalues and eigenvectors (covariance modes) have been determined based on the covariance matrix of chordwise fluctuating pressures. Energy contribution of the covariance modes is investigated. Obviously, the first covariance mode contributes dominantly 76.92%, 65.29%, 43.77% corresponding to models B/D=1, B/D=1 with S.P and B/D=5 to total energy of the pressure field system. If the first two modes are taken into account, the energy of these modes holds up to 90.19%, 86.26%, 65.79% of total energy. Uncorrelated principal coordinates associated with the covariance modes has been calculated from the measured pressure data, as first four principal coordinates of model B/D=1 (similar to other models, but omitted due to sake of brevity) at the flow case 1 and their corresponding power spectra are shown in Figure 2. It is noteworthy that the first principal coordinates not only dominate in the power spectrum but contain frequency characteristics of the random pressure field, whereas the other coordinates do not contain these frequencies. Thus, the first covariance modes and associated principal coordinate will play very important role in the identification of random pressure field due to their dominant energy contribution and frequency containing of hidden physical events of system.

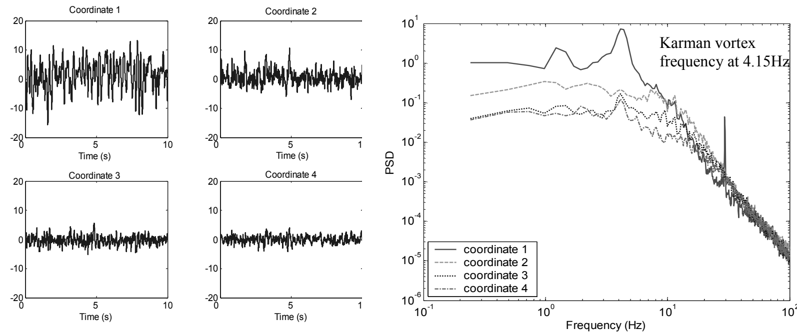

Fig.2 First four principal coordinates and their power spectra (B/D=1)

Secondly,frequency-dependant eigenvalues and eigenvectors are obtained from the cross spectral matrix of the observed pressure field. Figure 3 shows first five spectral eigenvalues of model B/D=1 (similar to other models, but omitted due to sake of brevity) on frequency band 0÷50Hz at the flow case 1. Clearly, the first spectral eigenvalues exhibit much dominantly than others, especially theses first eigenvalues also contain all frequency peaks of the pressure field, whereas others do not hold theses peaks. The first spectral modes of the chordwise fluctuating pressure fields of experimental model B/D=1 in the flow case 1 are also shown in Figure 3 in frequency band 0-50Hz. Energy contributions of spectral modes are expressed in Table 2. Similar to the covariance modes, the first spectral modes contain dominantly the system energy, for example, the first mode contribute 86.04%, 81.30%, 74.77%, respectively to the three experimental models at the flow case 1 (U=3m/s), the first two modes contribute almost 94.12%,91.45%,87.45% on total energy, respectively.

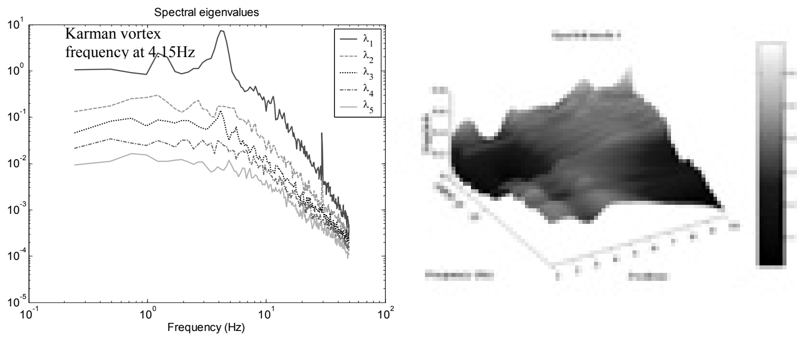

Fig.3 First five spectral eigenvalues and first spectral mode (B/D=1)

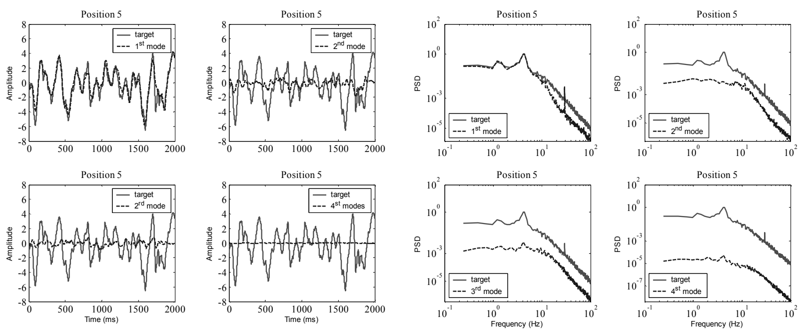

Fig.4 Effect of modes on pressure reconstruction (B/D=1)

Finally, effects of basic and cumulative modes on order-reduced reconstruction of the pressure field as well as role of the first mode on the field identification are investigated using both the proper transformation branches. Figures 4 shows the pressure reconstruction using individual covariance modes (1st mode, 2nd mode, 3rd mode and 4th mode) with verifying their spectral contribution to original pressure (as target). Reconstructed pressure using the first mode is close to the original pressure, especially its containing of dominant frequencies can be used to identify hidden physical characteristics of the original pressure field.

5. Conclusion

Significant role of the first covariance mode and the first spectral mode has been verified. The first mode contains certain frequency peaks of hidden physical phenomena, moreover, it contributes dominantly on the field energy. Thus the first mode is accuracy enough to reconstruct and identify the pressure field for many cases. The more complicated the pressure field distributes and the bluff body flow behaviors, the less important the first mode contributes and the more cumulative modes are needed to reconstruct the pressure field. In the comparison, the first spectral mode expresses the better than the first covariance mode in reconstructing the pressure field.

References

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