Evaluation of Branch Switching Characteristics in Coupled Flutter through Revised Step-by-Step Analysis

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1. **ABSTRACT –** Branch switching characteristics for coupled flutter of structural rectangular sections are analyzed using step-by-step flutter analysis (SBSA). Prior to the appearance of coupled flutter instability, the torsional branch (TB) controls the instability. However, after flutter onset both TB and heaving branch (HB) coexist, with sudden switching in particular velocity ranges. For lower velocity ranges, very good agreement can be obtained between conventional Complex Eigen Value Analysis (CEVA) and SBS method, but for higher velocity ranges this agreement cannot be attained. So SBSA is revised, leading to fairly good convergence between both methods.

2. **INTRODUCTION –** The advances in the modern Structural Engineering, triggered by the necessity of even longer span bridges, started a new era in the world of bridge constructions; novel design and construction techniques, associated to new girder cross-sections and materials, helped in this progress. The longer the spans become, the closer torsional and heaving natural frequencies of bridges get, making room for the arising of the 2-DOF coupled flutter instability, instead of the more common 1-DOF torsional flutter. In the wake of all this, aerodynamic phenomena not experienced by the bridges of the beginning of the last century became a major issue in the design of the long span bridges of the present days.

Since the famous failure of the old Tacoma Narrows Bridge in 1940, due to a torsional flutter instability in its main span, the investigations on flutter has progressively increased in number and in importance, and with the proposition of the flutter derivatives by Scanlan [1] the studies about the aerodynamic instability phenomena found a new route. Matsumoto et al investigated the inter-dependences between those derivatives [2], showing with the use of Step-by-Step Analysis (SBSA) the important role A_1^* and H_3 play for the occurrence of flutter [3].

Since the role a given aerodynamic derivate plays in the generation mechanism of flutter is dependent on what branch is governing the motion, the identification of the exact point of the branch switch is extremely important for establishing flutter stabilization strategies. So in this work the importance of using SBSA in this context is demonstrated, and a case study showing the differences produced by SBS, a revised SBSA and CEVA is presented, pointing the revised SBSA as an efficient tool for the precise identification of the branch switching point.

3. **BRANCH SWITCHING –** In the analysis of 2-DOF coupled flutter instability, the commonly used method is the Complex Eigen Value Analysis (CEVA), which provides means for the identification of the logarithmic damping, the flutter frequency, the amplitude ratio and the phase difference between torsional and heaving motions. However, this method is a mathematical approach for a 2-DOF dynamic instability, not based on the generation mechanism of the flutter itself, in a manner that some information is missed in the analysis process, e.g. branch switching. In order to fill this gap SBSA is used, and because this method is based on the flutter generation mechanism, the exact point for the branch switching can be identified.

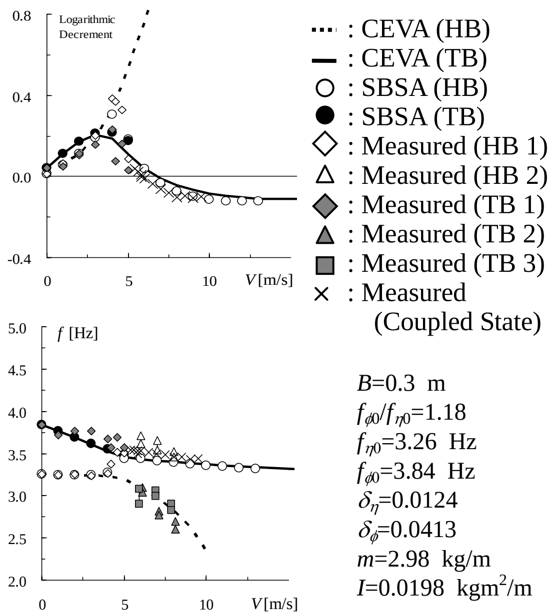

Figure 1 – Coupled flutter characteristics for *B/D* = 20 rectangular cylinder

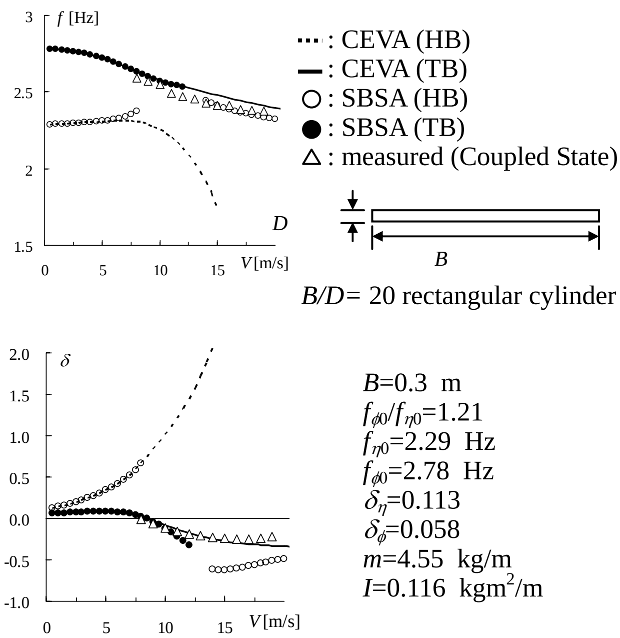

Figure 2 – Coupled flutter characteristics, *V-δ* and *V-f* characteristics of *B/D*= 20 rectangular cylinder

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Figure 1 shows the coupled flutter characteristics for a B/D=20 (B: chord length, D: depth) rectangular cylinder obtained through CEVA and SBSA, compared against free vibration tests, in which only SBSA shows the branch switch characteristics from TB to HB. At low velocity range both analyses show fairly good agreement, however a significant difference can be observed at high velocity range, after flutter onset, especially in the logarithmic damping characteristics, as shown in Figure 2.

4. **REVISED SBSA –** The causes of these numerical differences are related to the fact that the damping obtained from SBSA differs from the one obtained from free vibration tests. So SBSA was revised taking this in consideration, and in the first step the torsional motion was modified to the following equation:

$$\phi = \phi_0 e^{-\zeta_F \omega_F t} \sin\omega_F t \tag{1}$$

In the second step, heaving motion is induced by torsional motion as a forced vibration, with a certain amplitude ratio and phase difference. As a consequence, in the third step, this heaving motion induces the torsional oscillation. Finally, the flutter logarithmic damping and the flutter circular frequency are calculated with the following equations:

$$\delta_F = -\pi\Omega_1 A_2^* - \pi\Omega_1\Omega_2 \left\{ A_1^* \left|H_2^*\right| (1+\zeta_F{}^2)^{1/2} (\zeta_F \sin\theta_1 + \cos\theta_1) + A_1^* \left|H_3^*\right| (\zeta_F \sin\theta_2 + \cos\theta_2) - A_4^* \left|H_2^*\right| (1+\zeta_F{}^2)^{1/2} \sin\theta_1 - A_4^* \left|H_3^*\right| \sin\theta_2 \right\} \tag{2}$$

$$\omega_F = \left[\left\{ \omega_{\phi 0}{}^2 - \Omega_1 \omega_F{}^2 A_3^* - \Omega_1\Omega_2\omega_F{}^2 \{ A_1^* \left|H_2^*\right| (1+\zeta_F{}^2)^{3/2} \sin\theta_1 + A_1^* \left|H_3^*\right| (1+\zeta_F{}^2) \sin\theta_2 + A_4^* \left|H_2^*\right| (1+\zeta_F{}^2)^{1/2} (\cos\theta_1 - \zeta_F \sin\theta_1) + A_4^* \left|H_3^*\right| (\cos\theta_2 - \zeta_F \sin\theta_2) \} \right\} \right]^{1/2} \tag{3}$$

where, $\Omega_1 : (\frac{\rho b^4}{I}),\ \Omega_2 : (\frac{\rho b^2}{m}) \omega_F{}^2 \Big/ \sqrt{(\zeta_F{}^2 \omega_F{}^2 - \omega_F{}^2 - 2\zeta_\eta^* \zeta_F \omega_\eta^* \omega_F + \omega_\eta^{*2})^2 + (2\zeta_\eta^* \omega_\eta^* \omega_F - 2\zeta_F \omega_F{}^2)^2}$ (4)

Similarly, in HB, heaving motion is assumed as follows in step 1: $\eta = \eta_0 e^{-\zeta_F \omega_F t} \sin\omega_F t$ (5)

And then, the flutter characteristics are calculated with the following equations;

$$\delta_F = -\pi\Omega_1 H_1^* - \pi\Omega_1\Omega_2 \left\{ H_2^* \left|A_1^*\right| (1+\zeta_F{}^2)^{1/2} (\zeta_F \sin\theta_1 + \cos\theta_1) + H_2^* \left|A_4^*\right| (\zeta_F \sin\theta_2 + \cos\theta_2) - H_3^* \left|A_1^*\right| (1+\zeta_F{}^2)^{1/2} \sin\theta_1 - H_3^* \left|A_4^*\right| \sin\theta_2 \right\} \tag{6}$$

$$\omega_F = \left[\left\{ \omega_{\eta 0}{}^2 - \Omega_1 \omega_F{}^2 H_4^* - \Omega_1\Omega_2\omega_F{}^2 \{ H_2^* \left|A_1^*\right| (1+\zeta_F{}^2)^{3/2} \sin\theta_1 + H_2^* \left|A_4^*\right| (1+\zeta_F{}^2) \sin\theta_2 + H_3^* \left|A_1^*\right| (1+\zeta_F{}^2)^{1/2} (\cos\theta_1 - \zeta_F \sin\theta_1) + H_3^* \left|A_4^*\right| (\cos\theta_2 - \zeta_F \sin\theta_2) \} \right\} \right]^{1/2} \tag{7}$$

where, $\Omega_1 : (\frac{\rho b^2}{m}),\ \Omega_2 : (\frac{\rho b^4}{I}) \omega_F{}^2 \Big/ \sqrt{(\zeta_F{}^2 \omega_F{}^2 - \omega_F{}^2 - 2\zeta_\phi^* \zeta_F \omega_\phi^* \omega_F + \omega_\phi^{*2})^2 + (2\zeta_\phi^* \omega_\phi^* \omega_F - 2\zeta_F \omega_F{}^2)^2}$ (8)

So by using this revised SBSA, the previous analysis was redone, showing this time a fairly good agreement between revised SBSA, CEVA and wind tunnel tests, Figure 3.

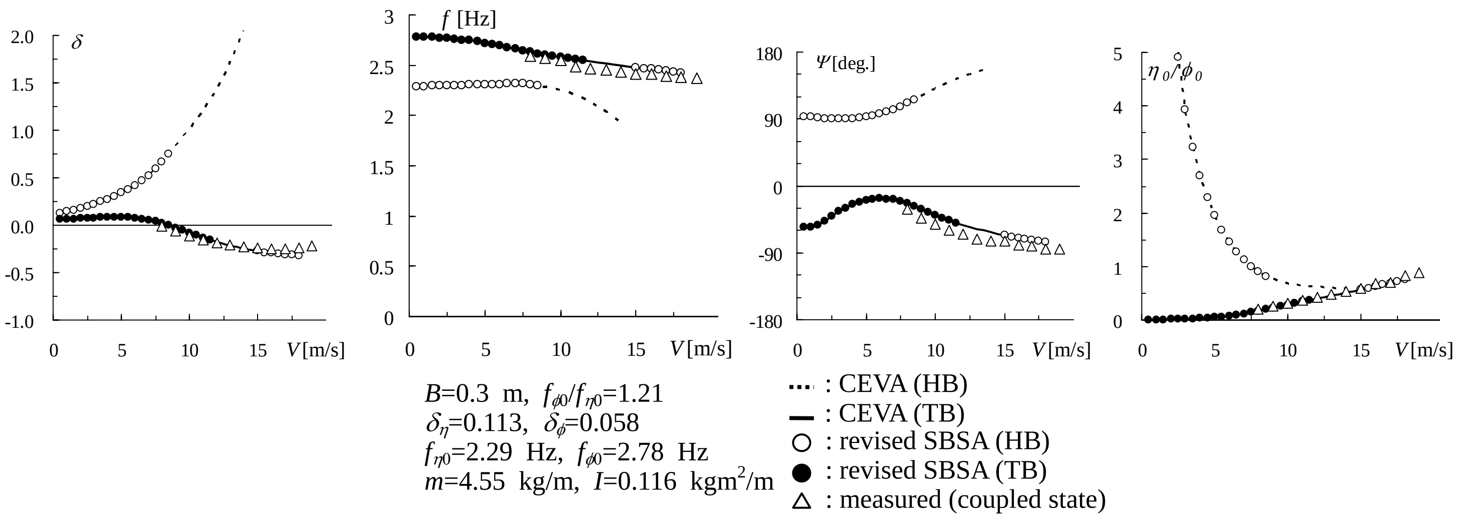

Figure 3 – Results of revised Step-by-Step analysis of B/D= 20 rectangular cylinder

5. **CONCLUSION –** SBSA is revised by considering the damping, and the results of revised SBSA show completely good agreement with those of CEVA. Besides, revised SBSA has advantage because the branch is identified based upon coupled flutter generation mechanism.

6. **REFERENCE –** 1) SCANLAN, R.H.; TOMKO, J.J., 1971 ***"Airfoil and Bridge Deck Flutter Derivatives"***, *Journal of Engineering Mechanics Divisions, ASCE,, 97, EM6;* 2) MATSUMOTO, M.; KOBAYASHI, Y.; NIIHARA, Y.; SHIRATO, H.; HAMASAKI, H. "***Flutter Mechanism and its Stabilization of Bluff Bodies"***. *Proceedings of 9ICWE.* Nova Delhi, India, 1995; 3) MATSUMOTO, M.; KOBAYASHI, Y.; SHIRATO, H. "***The Influence of Aerodynamic Derivatives on Flutter***". *Journal of Wind Engineering and Industrial Aerodynamics.* Amsterdam: Elsevier Science Publishers, v.60 pp.227-239, 1996.