Bridge dynamic characteristic estimation using canonical -form realization for ambient vibration

〇Bashir Ahmad JAWAID Graduate student Member Nagasaki University

Takatoshi OKABAYASHI Professor Fellow Nagasaki University

Toshihiro OKUMATSU Research Associate Member Nagasaki University

1. Introduction

In this study AR and ARMA model parameters calculation method has been addressed using canonical form realization based on Hankel matrix. This research is verification for effectiveness of the proposed method since canonical realization method is simple and faster than the previous method for calculation of parameters. Estimation accuracy of the method was examined by comparison to FEM method. In this study attention has been paid to accuracy of frequency, damping and mode shapes.

2. Studied Model

Langer bridge is selected as the object model bridge, and **Fig. 1** shows its general view. **Table 1** shows the model bridge element characteristics. **Table 2** shows the natural frequency result of the model bridge from 1st to 8th order by FEM method. **Fig. 2** shows the vibration modes from 1st to 8th order which obtained by FEM method.

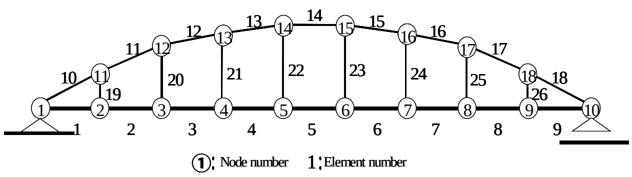

Fig. 1 General view of Langer Bridge

Table 1. Model bridge characteristics

Characteristics	Units	Numerical value
Effective span	L (m)	58.995
Model Height	H(m)	9.36
Density of steel	D (t/m^3)	7.85
Young's Modulus	E (t/m^2)	2.1*10^7
Total mass of the model	Ton	19.3
Total weight of model	Ton	189.17
Cross-section area	A1 (m^2)	0.0224
	A2 (m^2)	0.0123
	A3 (m^2)	0.0137
	A4 (m^2)	0.006015

Table 2. Natural frequency (Hz)

1st	1.749
2nd	2.893
3rd	5.69
4th	7.375
5th	9.416
6th	11.179
7th	14.236
8th	15.615

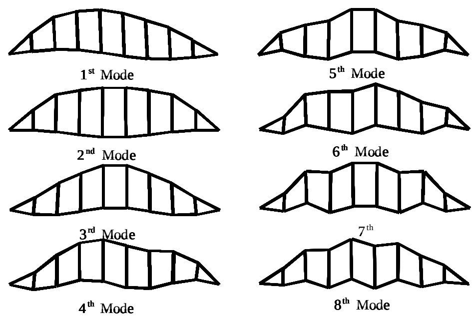

Fig. 2 Mode shapes

3. Methodology

Discretized representation of motion equation as following

$$\boldsymbol{x}(k+1)=\overline{\boldsymbol{A}}\boldsymbol{x}(k)+\overline{\boldsymbol{B}}\boldsymbol{f}(k) \quad (1a)$$

$$\boldsymbol{y}(k)=\overline{\boldsymbol{C}}\boldsymbol{x}(k) \quad (1b)$$

Where $\overline{\boldsymbol{A}}=e^{Ah}$, $\overline{\boldsymbol{B}}=(e^{Ah}-\boldsymbol{I})\boldsymbol{A}^{-1}\boldsymbol{B}$, $\overline{\boldsymbol{C}}=\boldsymbol{C}$.

State space and discretized representation of motion equation in terms of generalized observational matrix yields to transformation of motion equation into multidimensional ARMA model. A multi-dimensional ARMA model can represent by a finite multi-dimensional AR model, and following equation can show a multi-dimensional AR model.

$$\boldsymbol{y(k)}+\sum_{s=1}^{p}\boldsymbol{G}_s\boldsymbol{y(k-l)}=\boldsymbol{e(k)} \quad (2)$$

Moreover, if the observed value of m point as a m dimension vector, then variance and covariance matrices of the observed values can be shown as following equation.

$$\boldsymbol{\Lambda}(\mathbf{l})=\frac{1}{N}\sum_{k=1}^{N}\mathbf{y}(\mathbf{k})\mathbf{y}^{\mathrm{T}}(\mathbf{k-l}),\quad \boldsymbol{\Lambda}(-\mathbf{l})=\frac{1}{N}\sum_{k=1}^{N}\mathbf{y}(\mathbf{k})\mathbf{y}^{\mathrm{T}}(\mathbf{k}+\mathbf{l}) \quad (3)$$

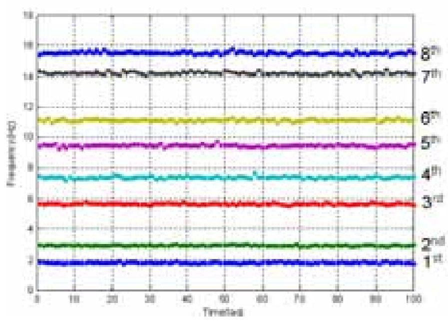

Fig. 3 Frequency (Hz)

Keyword: Canonical realization ARMA model Dynamic characteristics FEM Ambient vibration

連絡先 長崎大学工学部(〒852−8521 長崎市文教町 1−14，Tel 095-819-2615，Fax 095-819-2627)

The following relation can be derived from observerability matrix.

$$\mathbf{P}_p\mathbf{A}^{s+1}=\begin{bmatrix}\mathbf{CA}^{s+1}\\ \mathbf{CA}^{s+2}\\ \vdots\\ \mathbf{CA}^{s+p}\end{bmatrix}=\begin{bmatrix}\mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ -\mathbf{G}_p & -\mathbf{G}_{p-1} & -\mathbf{G}_{p-2} & \cdots & -\mathbf{G}_1\end{bmatrix}\begin{bmatrix}\mathbf{CA}^{s}\\ \mathbf{CA}^{s+1}\\ \vdots\\ \mathbf{CA}^{s+p-1}\end{bmatrix}=\hat{\mathbf{A}}\mathbf{P}_p\mathbf{A}^{s} \qquad (4)$$

Right – multiplying $\mathbf{Q}_q$ to both sides of eq. (4) yields.

$$\mathbf{P}_p\mathbf{A}^{s+1}\mathbf{Q}_q=\hat{\mathbf{A}}\mathbf{P}_p\mathbf{A}^{s}\mathbf{Q}_q \qquad (5)$$

Eq. (5) in condensed form as following

$$\mathbf{H}(s)=\hat{\mathbf{A}}\mathbf{H}(s-1) \qquad (6)$$

Where H(s) and H(s-1) are Hankel matrix. In the detailed form the eq. (6) can be shown as follows

$$\begin{bmatrix}\Lambda(s+1) & \cdots & \cdots & \Lambda(s+q)\\ \vdots & & & \\ \vdots & & & \\ \Lambda(s+p) & \cdots & \cdots & \Lambda(s+p+q)\end{bmatrix}=\hat{\mathbf{A}}\begin{bmatrix}\Lambda(s) & \cdots & \cdots & \Lambda(s+q-1)\\ \vdots & & & \\ \vdots & & & \\ \Lambda(s+p-1) & \cdots & \cdots & \Lambda(s+p+q-2)\end{bmatrix} \qquad (7)$$

The system matrix $\mathbf{A}$ can be obtained by two methods. First method to solve eq. (7) i.e. $\hat{\mathbf{A}}$ becomes similar form of $\mathbf{A}$.

The second method is to calculate the system matrix $\hat{\mathbf{A}}$ from extracted lower blocks of eq.(7) which are similar to $\mathbf{G}=\lfloor -\mathbf{G}_1 \quad -\mathbf{G}_2 \quad \cdots \quad -\mathbf{G}_p \rfloor$ in the eq.(4), and becomes similar to the Yule-Walker equation, Which is as set of linear equations relating the parameters of an AR model with the auto correlation sequence in the matrix form.

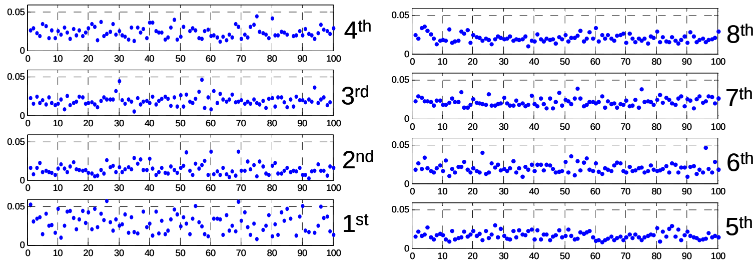

Fig. 4 Estimated modal damping

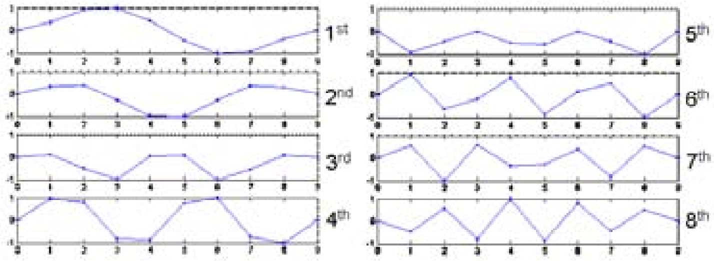

Fig.5 Estimated mode shapes

Table 3. Dynamic characteristics estimation simulation result for Langer bridge .

Mode order	Frequency					Modal damping				
	Analytical value	Mean value	Percentage of error	Standard deviation	Coefficient of variation	Analytical value	Mean value	Percentage of error	Standard deviation	Coefficient of variation
1st	1.749	1.802	3.03	0.0329	1.8234	0.02	0.032	60.00	0.0163	49.4033
2nd	2.893	2.921	0.97	0.0276	0.9432	0.02	0.015	25.00	0.007	45.4519
3rd	5.69	5.647	0.76	0.0439	0.7768	0.02	0.021	5.00	0.0078	37.4129
4th	7.375	7.399	0.33	0.0597	0.8065	0.02	0.025	25.00	0.0094	36.4893
5th	9.416	9.434	0.19	0.0577	0.612	0.02	0.018	10.00	0.0049	28.1424
6th	11.179	11.123	0.50	0.0612	0.5505	0.02	0.021	5.00	0.0064	30.3838
7th	14.236	14.225	0.08	0.0814	0.5723	0.02	0.023	15.00	0.0054	23.4892
8th	15.615	15.523	0.59	0.0776	0.5001	0.02	0.021	5.00	0.0051	23.8273

4. Results

Graphical representation of frequencies and modal damping are shown in **Fig. 3** and **Fig**. **4** respectively. **Fig. 5** shows modes of vibration of estimated data and analysis result for all observed points. **Table 3** shows the dynamic characteristics estimation result for mean value, percentage of error, standard deviation and coefficient of variation for frequency and modal damping. The percentage of error is low for frequency as well as for modal damping. Considering, the accuracy, the obtained data shows; the values are close for eigenvalue analysis and estimated dynamic characteristics where the margin of discrepancy is small for frequency as well as for modal damping. The percentage of error for frequency is the lowest for the 5th mode and the highest is for the 1st mode. The percentage of error for the damping has the lowest value for the 3rd, 6th and 8th modes and the 1st has the highest value.

5. Conclusions

The proposed method is competent to calculate the Yule Walker equation simply and fast .Obtained result showed good accuracy. Comparison of proposed method result matched for all of the modal shapes frequencies and modal damping. The margins of the discrepancy were small. A unified procedure has been presented in this paper. The proposed method has been applied to real bridge monitoring but due to the space limit could not integrated in this paper thus the results for real bridge ambient vibration test will be presented at the time of presentation.

References

[1] Jimin He and Zhi-Fang Fu: Modal Analysis, BH Press Great Britain, 2001. [2] Jer-Nan Juang: Applied System Identification, Prentice Hall PTR, 1994. [3] O.C.Zienkiewicz, R.L.Taylor: The finite element method, Elsevier, 2005.