Flexible Solid and Fluid Interaction Analysis

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1. Introduction

The most famous solid-fluid interaction has a Lagrangian solid and an Eulerian fluid model, which usually implies two solvers which interact during the simulation. The interaction algorithm for above mentioned computational framework can be however computational expensive to find the contact domain. With the present approach, this problem vanishes since such kind of interaction algorithm is not required. The interaction or contact simulation is the concept of an element mixture theory. More than one material are than handled in an element. Only an Eulerian mesh is used, so only one solver.

2. Computational Framework

The solution of present fixed-mesh finite element method[1] is splitted here in a Lagrangian step and an advective step. Later step transports the material and corresponding solution variables through the remapped mesh. Both steps, the Lagrangian and Eulerian are solved at each time step. The flowchart of a computational step is described in below box. After reading and initializing the program data, a loop is started until the wished end time is reached.

- **Lagrangian step**
  - + calculate element stress
    - - solid stress
    - - fluid stress
    - - average stress
  - + calculate internal nodal forces
  - + calculate nodal accelerations
  - + calculate nodal velocities
    - - calculate fluid pressure
    - - update nodal velocities
  - + calculate coordinates
- **Eulerian or advective step**
  - + advect solution variables
  - + advect nodal velocities

It is worth to mentioned that implemented subroutines integrate the element with a single gauss point, which simplify the interaction of solution variables between solid and fluid.

3. Navier-Stokes Equations

The primitives variables of an incompressible viscous fluid dynamic process are mainly given by following the Navier-Stokes equations, namely, the momentum equation and the incompressibility constrain equation.

$$\rho\dot{\boldsymbol{v}} + \nabla p = \mu\nabla^2\boldsymbol{v} + \rho\boldsymbol{b} \tag{1}$$

$$\nabla \cdot \boldsymbol{v} = 0 \tag{2}$$

where ρ is the fluid density, $\boldsymbol{v}$ is the velocity field, p is the pressure, μ is the dynamic viscosity and $\boldsymbol{b}$ is the body acceleration. A way of decoupling velocity and pressure is the concept of a fractional step method, which splits the momentum equation in

$$\rho\dot{\boldsymbol{v}} = \nu\nabla^2\boldsymbol{v} + \rho\boldsymbol{g} \tag{3}$$

$$\rho\dot{\boldsymbol{v}} = -\nabla p \tag{4}$$

The concept of the fractional method is carried out in three steps:

1. the calculation of an intermediate velocity (neglecting pressure effects)

$$\boldsymbol{v}_*^{n+1} = \boldsymbol{v}^n + \frac{\Delta t}{\rho}\left[\nu\nabla^2\boldsymbol{v}^n + \rho\boldsymbol{g}\right] \tag{5}$$

2. the calculation of the pressure by considering the previously calculated intermediate velocity. Taking the divergence or curl ($\nabla\cdot$) of Eq. (4) and considering the incompressibility constrain of Eq. (2), the pressure poisson equation is reached

$$\nabla \cdot \boldsymbol{v}^{n+1} = \nabla \cdot \boldsymbol{v}_*^{n+1} - \frac{\Delta t}{\rho}\nabla \cdot \nabla p \tag{6}$$

$$\nabla^2 p = \frac{\rho}{\Delta t}\left[\nabla \cdot \boldsymbol{v}_*^{n+1}\right] \tag{7}$$

3. the correction of the velocity by considering pressure effects

$$\boldsymbol{v}^{n+1} = \boldsymbol{v}_*^{n+1} - \frac{\Delta t}{\rho}\nabla p \tag{8}$$

4. Solid-Fluid Interaction

Mixture theories concept the treatment of more than one material in an element. The simplest mixture theory, also called the mean strain rate mixture theory, is favorized because of its simplicity and robustness [2]. Here, we used it for a solid-fluid interaction problem. The simplest mixture theory have the assumption that the strain rate $\dot{\boldsymbol{\epsilon}}$ is same for all present materials in an element. So the stress rate $\dot{\boldsymbol{\sigma}}_m$ of material m is given by

$$\dot{\boldsymbol{\sigma}}_m = \mathbb{C}_m : \dot{\boldsymbol{\epsilon}} \tag{9}$$

where $\mathbb{C}_m$ is the constitutive tensor of material m.

The updated stress $\boldsymbol{\sigma}_m^{n+1}$ by time $n+1$ is then

$$\boldsymbol{\sigma}_m^{n+1} = \boldsymbol{\sigma}_m^{n} + \dot{\boldsymbol{\sigma}}_m \Delta t \tag{10}$$

The element mean stress $\overline{\boldsymbol{\sigma}}^{n+1}$ is then

$$\overline{\boldsymbol{\sigma}}^{n+1} = \sum_{m=1}^{nmat} \boldsymbol{\sigma}_m^{n+1} f_m \tag{11}$$

where $nmat$ is the number of materials present in the element and f_m is the element density function of each material m.

5. Computational Results

In following calculation, an elastic ring falls free onto a fluid. For simulating a certain impact velocity when the free-falling ring reaches the fluid, an initial velocity of 0.2m/s downwards has been added to the elastic ring. Material properties are shown in Table 1.

Table1 Material data: ring-fluid

ring	
Density [kg/m^3]	500
Young modulus [MPa]	0.05
Poisson's ratio [-]	0.30
fluid	
Density [kg/m^3]	1000
Dyn. viscosity [N.s/m^2]	0.001

Fig. 1 shows the computational model, 100×66 elements. While the lower part of the ring is supported by the fluid at impact, the upper part bears a downward forcing due to gravity, 9.81m/s. The result of this force combination forms an oval shape. Thanks the elasticity of the ring it tries to recover its initial shape as shown in the simulation plots in Fig. 2. Xiao and Yabe [3] published a similar simulation.

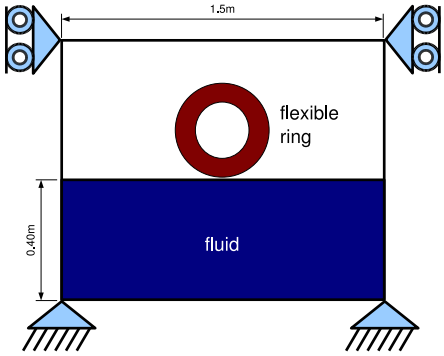

Figure1 Ring Falling onto Fluid

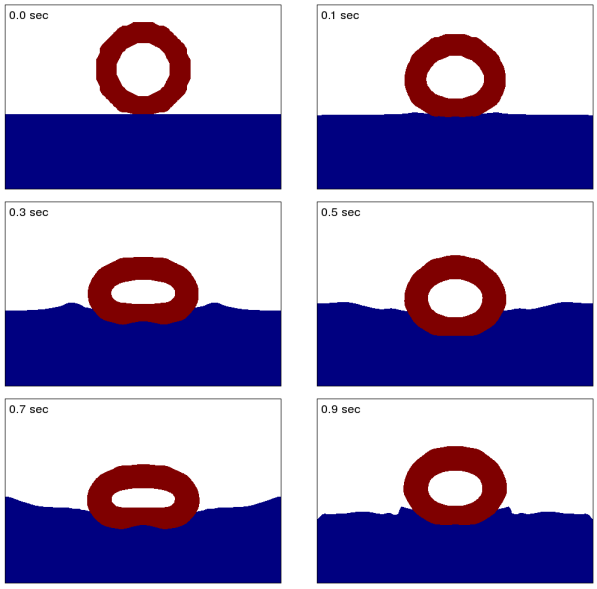

Figure2 Computational result of ring falling onto fluid

6. Conclusion

A Solid-fluid interaction test with a flexible or elastic body has been selected rather than a rigid body. Interaction is well simulated as shown in the ring-fluid calculation. Buoyancy effects are also well simulated. In fact, calculations show promising results regarding the use of simple mixture theory. A complete stress mixture is carried out at each element rather than an interaction algorithm is used.

References

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