Energy Extraction by Utilizing the Pitching Motion of a Floating Vessel

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1. Introduction

An energy extraction system is proposed here, which extracts the energy from a tidal flow or stream flow by utilizing the pitching motion of a floating vessel. To obtain the required oscillation of the vessel, a number of flat plates located at the base of the vessel are rotated at a constant angular velocity in the flow using an external power source. The time period of the rotation of the plates can be regulated through a speed controller bringing about a change in the amplitude of the oscillation. The compression and the suction of air in the vertical water chambers caused by the pitching motion can be utilized to drive Wells turbines located on either sides of the vessel. The pitching motion is discussed using the 1st order model that takes into account the effects of the vessel's inclination on the forces exerted. Finally, the dynamics of the plate rotation is discussed.

2. Dynamics of the Pitching Motion

The plates located at the bottom of the vessel experience a drag force from the flow which is proportional to the square of the flow velocity. This force is converted into a periodic force as the plates are rotated at a constant angular velocity. The moment produced by this force will cause a periodic oscillation the vessel about its transverse axis. Equation of the pitching motion can be written as

$$\left(I\frac{d^2\theta}{dt^2} + \rho g I_y \theta \right) = M_1 + M_2 \qquad (1)$$

where, I: mass moment of inertia of the vessel, I_y: moment of inertia of the cross-section of the vessel about the transverse axis at waterline, θ: pitching angle at any instant of oscillation, ρ: density of water, g: acceleration due to gravity.

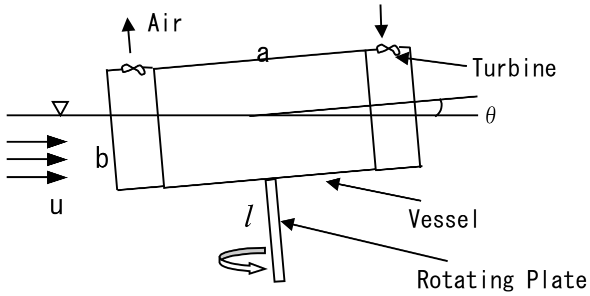

Fig. 1. Profile of the floating vessel

M_1 is the moment of the flow force experienced by the submerged area of the vessel about its transverse axis given as

$$M_1 = \frac{1}{2}\rho C_D b\left[\frac{1}{2}u^2\cos^2\theta(\sigma c/\rho)^2 - \frac{2}{3}u\cos\theta\frac{d\theta}{dt}(\sigma c/\rho)^3 + \frac{1}{4}\left(\frac{d\theta}{dt}\right)^2(\sigma c/\rho)^4\right] \qquad (2)$$

M_2 is the moment of the drag force experienced by the plates about the transverse axis, which is given as

$$M_2 = n\frac{1}{2}\rho C_D\frac{b}{m}\left|\cos(\omega t)\right|\cdot\left[\frac{1}{2}u^2\cos^2\theta\left\{(\sigma c/\rho)^2-(\sigma c/\rho)^2\right\}\right] -$$

$$\frac{2}{3}u\cos\theta\frac{d\theta}{dt}\left\{(\sigma c/\rho+l)^3-(\sigma c/\rho)^3\right\} + \frac{1}{4}\left(\frac{d\theta}{dt}\right)^2\left\{(\sigma c/\rho+l)^4 - l^4\right\} \qquad (3)$$

where, σ: density of the vessel, u: flow velocity, C_D: drag coefficient, ω: angular velocity of the plates, n: no. of plates, m: an integer number which correlates to the width of the vessel and the number of plates, a, b, c: length, width and height of the vessel as shown in Fig. 1. Combining equations (1), (2) and (3), we obtain,

$$\frac{d^2\theta}{dt^2} + \frac{4C_D}{a^3}u\cos\theta\left[(\sigma c/\rho)^2 + \frac{n\rho}{m\sigma c}\left|\cos(\omega t)\right|\left\{(\sigma c/\rho+l)^3-(\sigma c/\rho)^3\right\}\right]\left(\frac{d\theta}{dt}\right)$$

$$-\frac{3C_D}{2a^3}\left[(\sigma c/\rho)^3 + \frac{n\rho}{m\sigma c}\left|\cos(\omega t)\right|\left\{(\sigma c/\rho+l)^4-(\sigma c/\rho)^4\right\}\right]\left(\frac{d\theta}{dt}\right)^2 + \frac{\rho g}{\sigma c}\theta =$$

$$\frac{3C_D}{a^3}\left[(\sigma c/\rho) + \frac{n\rho}{m\rho c}\left|\cos\omega t\right|\left\{(\sigma c/\rho+l)^2-(\rho c/\rho)^2\right\}\right]u^2\cos^2\theta \qquad (4)$$

Equation (4) can be solved using numerical methods to obtain the time series of the pitching angle. Figs. 3 and 4

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show the results obtained from the Runge-Kutta method for the dimensions specified in Table. 1 at different periods of the plate rotation.

3. Dynamics of the plate rotation

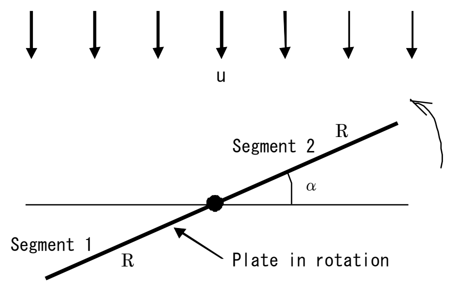

Fig. 2. Plan view of the rotating plate

Since the plates are rotated in a unidirectional flow, the work required to rotate them at a given time will depend on the angle of rotation at that instant. In other words, the work will be a function of time. The effect of the pitching motion on the rotation of the plates is ignored here, as the streamwise component of the plate's rotation is small compared to the flow velocity when narrow preferable plates are used as shown by Fig. 5. Equations for the work required to rotate the plates are given below.

From t = 0 to $t = \cos^{-1}(R\omega/u)/\omega$

$$P = \frac{2}{3}\rho C_D l\omega^2 R^3 u \cos\alpha \tag{5}$$

From $t = \cos^{-1}(R\omega/u)/\omega$ to $t = \pi/2\omega$

$$P = \frac{1}{2}\rho C_D l\omega\left(u^2\cos^2\alpha\cdot R^2 + \frac{1}{2}\omega^2 R^4\right) - \frac{1}{2}\rho C_D l\omega\left(u^2\cos^2\alpha\cdot R_1^2 - \frac{4}{3}u\cos\alpha\cdot\omega R_1^3 + \frac{1}{2}\omega^2 R_1^4\right) \tag{6}$$

From $\pi/2\omega$ to $t = \pi/\omega - \cos^{-1}(R\omega/u)/\omega$

$$P = \frac{1}{2}\rho C_D l\omega\left[u^2 R^2\cos^2(\pi-\omega t) + \frac{1}{2}\omega^2 R^4\right] - \frac{1}{2}\rho C_D l\omega\left[u^2 R_1^2\cos^2(\pi-\omega t) - \frac{4}{3}u\omega R_1^3\cos(\pi-\omega t) + \frac{1}{2}\omega^2 R_1^4\right] \tag{7}$$

And from $t = \pi/\omega - \cos^{-1}(R\omega/u)/\omega$ to $t = \pi/\omega$

$$P = \frac{2}{3}\rho C_D l\omega^2 u R^3\cos(\pi - \omega t) \tag{8}$$

where, P: work needed to rotate a plate, l: length of plate, R: half width of the plate, R_1: point on the plate where the flow velocity is equal to the tangential component of the plate rotation, α: angle of rotation at any instant, TR: time period of plate rotation.

Table. 1. Dimension of the vessel and plates

Physical Quantities		Value
Density of water	ρ (kg/m^3)	1000
Density of vessel	σ (kg/m^3)	700
Flow velocity	u (m/s)	2
Drag coefficient	C_D	1
Length of vessel	a (m)	10
Width of vessel	b (m)	7
Height of vessel	c (m)	6
Length of plates	l (m)	5
Number of plates	n	6

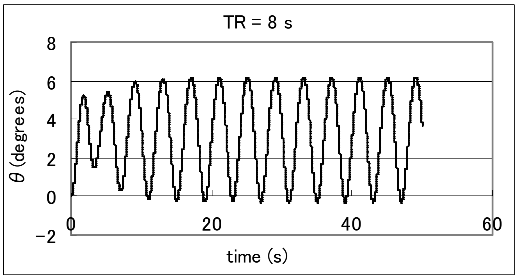

Fig. 3. Pitching angle at TR = 8 s

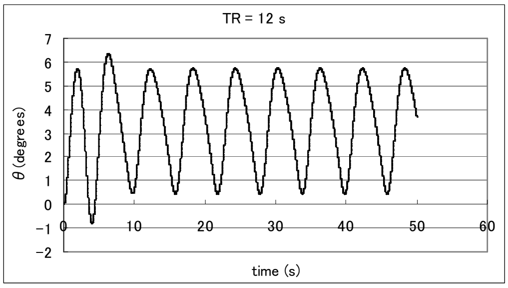

Fig. 4. Pitching angle at TR = 12 s

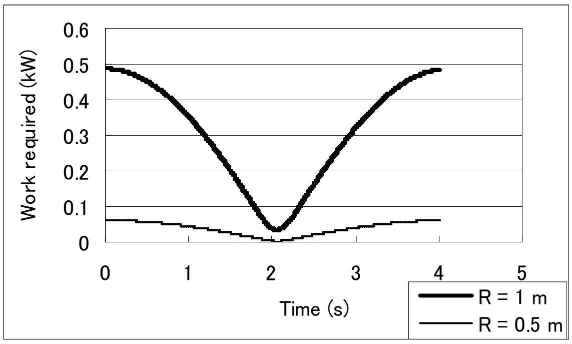

Fig.5. Work required to rotate a plate

4. Conclusion

Calculation of the pitching angle showed that the oscillation is maximum during resonance which occurs at around TR= 8 s for the given system. Equations of the work required to rotate the plate contain terms, which are proportional to the second, third, and fourth power of the width of the plates. Hence it is more economical to use many plates with small width than a few plates with large width.