Quantiles of Neyman-Scott Rectangular Pulse Rainfall Model for Hydrologic Designs

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1. Introduction

The use of stochastic models with available records to generate synthetic hydrologic data has been used widely in the field of water resources engineering. Such stochastic methods are used with Monte Carlo simulations in the event when record length is insufficient for effective hydrologic analysis. For rainfall data, several techniques are available such as the Poisson marks model (Rodriguez-Iturbe *et al.* (1987)), Poisson rectangular pulse models (Rodriguez-Iturbe et al. (1987)), and Clustered Poisson Rectangular pulse models (Rodriguez-Iturbe *et al.* (1987) and Burlando and Rosso, (1993)). The latter type is the key technique used in this study.

A Clustered Poisson Rectangular Pulse Rainfall Model (CPRPRM) is a stochastic technique in which rainfall is considered to be a system of cell clusters of random size, arrival time, intensity, and duration. In this study, statistical parameters were estimated from short rainfall data sets (10-15 years) to calculate parameter sets for the Neyman-Scott (NS) rectangular pulse model.

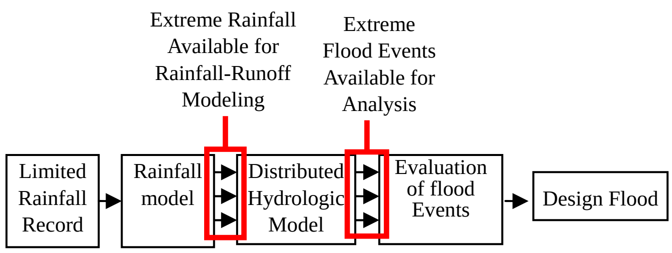

Fig.. 1 Concept of the Study *Quantiles of Neyman-Scott Rectangular Pulse Rainfall for Design Considerations.*

By using the NS model, it was possible to generate time series possessing several second-order properties (mean, standard deviation, autocorrelation coefficient order 1, etc.) consistent with the study area. In this study however, the prime concern was in determining the extreme values of this synthetic data and the determination of its acceptability. Leading applications of such information include design flood evaluation for small to medium conservation reservoirs (300-500 km^2) with minimal rainfall records. In succession to this study, the authors intend to apply the method for the evaluation of the design floods of the Kamiishiba (210km^2) and Asuwagawa (350km^2) River Basins in Japan. The schematic of the entire study is shown in Figure 1.

2. NS Rainfall Time Series Generation Method

The Neyman-Scott model consists of essentially five probability distributions. In this model, clusters of cells are linked integrally to a storm origin with mean occurrence rate λ, regarded as a Poisson process. Each storm can have a random number of cells described by a geometric distribution. Each cell has a corresponding independently random intensity and duration, both of which are identically characterized by the exponential distribution. A succinct representation of the four previously mentioned distributions can be written as:

$$P[C=c]=\frac{(1-1/\mu_c)^{c-1}}{\mu_c}$$
$$f(t_d)=\beta\exp(-\beta t_d)$$
$$f(i_c)=1/\mu_x\exp(-1/\mu_x i_c)$$
$$f(t_c)=\delta\exp(-\delta t_c)$$

where:

P[C=c] = probability that the number of cells of a storm C is equal to c.

μ_c = mean number of cells in a storm.

$f(t_d)$ = probability that the arrival of a cell from the storm origin is t_d.

$1/\beta$ = mean displacement of a cell from the storm origin

$f(i_c)$ = probability that the intensity of a cell is equal to i_c.

μ_x = mean intensity of a cell.

$f(t_c)$ = probability that the duration of a cell's life is equal to t_c.

$1/\delta$ = mean cell life span.

In a succeeding study, an alternative CPRPRM, the Bartlett-Lewis (BL) rectangular pulse model, will be used for the same objectives. The Bartlett-Lewis model is similar in framework and is not shown here for brevity. Both methods make use of Poisson process-based arrival of rainfall, with assumed exponentially displaced origins

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or rain clusters and storm origin. In the NS process, the positions of the location of the rainfall distribution are identically distributed random variables. In the BL process, the interarrival times of successive cells are independent identically distributed random variables.

3. Parameter Estimation

From the method of moments, the NS and BL parameters can be linked to actual record data statistics. In either model, parameter estimation involves a small system of nonlinear transcendental equations that are solved using an unconstrained minimization technique. In this study, two methods are considered: the downhill simplex method, and the genetic algorithm. In the downhill simplex method, a simplex is initiated from a set of N+1 initial guesses from an objective function of N independent variables (in this case, the parameters of either the NS or BL processes). This simplex then steadily readjusts its shape in the solution domain until a minimum is determined. This method was used in this study due to its compactness and independence of derivatives (unlike the gradient based methods such as the method of steepest descent, Powell's method, etc.). However, in the event when the precision required is very high, the downhill simplex method may prove too slow to converge. For this reason, the genetic algorithm was also considered.

4. CPRPRM Model Implementation

Figure 2 is a sample realization rainfall for Kamishiiba generated from the Neyman-Scott model. As each model is a based on probability distributions of some aspect of the temporal distribution of the rainfall, a random number generator was necessary to use these equations. Uniform deviates were used to seed the probability distributions of namely the storm arrival, number of cells, cell arrival, rainfall intensity, rainfall duration for the NS process, and the storm arrival, number of cells, cell interarrival times, rainfall intensity, and duration for the BS process. Thus, multiple realizations of the rainfall process can be generated for the study areas.

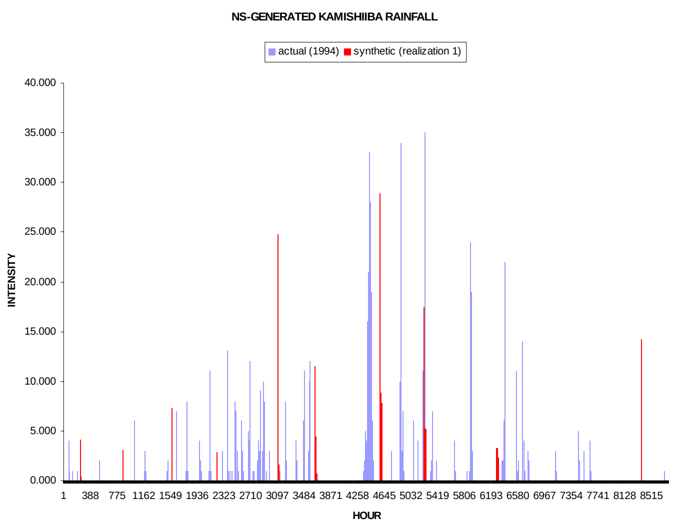

Fig.2. Sample realization hyetograph generated from the Neyman-Scott Rectangular Pulse Rainfall Model.

5. Further Studies

Halfway through its completion, this study requires several components. Upon the generation of synthetic rainfall records, quantiles are evaluated in the following procedure. For each realization year (say of 100 years), the maximum hourly value of rainfall is recorded. A histogram is generated for the entire record of maximums. A candidate parametric fit is tested for its ability to completely represent the histogram. The distribution with the best fit is then used to determine the quantile values (based on 90%, 95% or higher excedance probability). In addition, a nonparametric method to determine the quantiles may also be employed. Such methods include the jackknife and the bootstrap estimate of quantiles.

In this study, the OHYMOS library will be utilized (i.e.: KSEDGE.exe) to determine streamflow from the synthetic rainfall generated. This model uses the Kinematic Wave technique to generate runoff. Each quantile realization rainfall can thus correspond to a realization streamflow.

As shown in Figure 1, the study essentially consists of model parameter approximation by the previously mentioned unconstrained minimization methodology, actual random number seeding and physically based modeling by using an existing external system (i.e.: the OHYMOS Library).

Reference:

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