DESIGN FORMULAS OF DAMPERS FOR VIBRATION CONTROL OF STAY CABLES

The University of Tokyo, Member (VCA), HOANG Nam
The University of Tokyo, Fellow (JSCE), FUJINO Yozo
Nippon Steel Corporation Member (JSCE), SHINSUKE Yamazaki

ABSTRACT

Performance of dampers in controlling large-amplitude vibration of stay cables in bridges depends on many parameters in which the sag, inclination and bending stiffness of the cable, as well as the effect of stiffness of anchor tube have significant influences. In this study, simple formulas of the modal damping ratio of the cables with attached dampers are analytically derived considering the sag in associated with inclination and stiffness of anchor tube. Other types of dampers encountered in practice e.g., high-damping rubber devices is also analyzed. Also the combination of two dampers, each near one end of the cable, is shown to proportionally raise the modal damping level.

1. INTRODUCTION

Performance of a damper in suppressing large-amplitude vibration of stay cable is often evaluated in terms of the modal damping raised after the damper is added. This renders a complex eigenvalue analysis of cable with damper from which the modal damping value of an individual mode is determined by the imaginary part of the eigenfrequency. Kovacs (1982) first identified the existence of an optimal size for a transverse added viscous damper. After that, Yoneda and Maeda (1989) and Uno et al. (1991) have conducted numerical studies on the optimum damper size. Notable is the work of Pacheco et al. (1993) who have obtained a universal estimation curve relating the modal damping ratio to the damper size. In a different approach, Krenk and his associates have utilized a small perturbation on well-known solutions of cable without damper to derive analytical solutions for the problems of a horizontal cable with a transverse damper. In 2000, Krenk successfully derived a simple analytical formula for the numerical results of Pacheco et al. Further the analytical solutions have been extended to consider the influence of the cable sag (Krenk and Nielsen, 2002). In the present study the damping effect of a transverse damper on a general inclined cable with sag is investigated, considering the anchor tube stiffness of internal dampers. The study aims to derive simple formulas relevant to the design of a damper for stay cables considering influencing factors. The remains of the paper addresses the effect of a practical damper device using high-damping rubber (HDR), and the combined use of two dampers, one viscous damper at the lower end of the cable and one HDR damper at the other end.

2. DESIGN OF AN INCLINED SAG CABLE WITH A DAMPER

Consider an inclined cable with a transverse damper attached at location x_c from the lower end (Fig. 1). The cable has mass per unit length m, chord length L, and is inclined at angle θ to the horizontal. Assume that the cable tension is large enough so that the static profile of the cable can be accurately described by the parabola. If g denotes gravity acceleration; $H = T_h/\cos\theta$ is the chord tension; T_h is the horizontal component of cable tension, the sag at mid-span is given by Irvine (1981):

$$d = \frac{mgL^2\cos\theta}{8H} \quad (1)$$

Krenk and Nielsen (2002) have derived analytical asymptotic results for a horizontal cable with sag. For an inclined cable, since only the gravity component perpendicular to the chord is involved (Eq. 1), the steeper the chord inclination the lesser the sag of the cable. From their works, the damping ratio of an individual mode n can be evaluated by an accurate approximation re-written here as

$$\frac{\xi_n}{x_c/L} \cong R_s \,\mathrm{Im}\left[\frac{x_c F_c(\beta_n^{s0})/H\tilde{v}_c}{1 + x_c F_c(\beta_n^{s0})/H\tilde{v}_c}\right] \quad (2)$$

where R_s is the reduction factor due to influence of cable sag.
In case of a viscous damper, $f_c(t) = c\,\partial v(x_c,t)/\partial t$ or $F_c = ic\omega\tilde{v}_c$,

$$\frac{\xi_n}{x_c/L} = R_s\frac{\eta_n}{1+\eta_n^2}, \quad n = 1, 3, \ldots \quad (3)$$

where $\eta_n = \beta_n^{s0} x_c\, c/\sqrt{Hm}$ is a non-dimensional damper parameter.

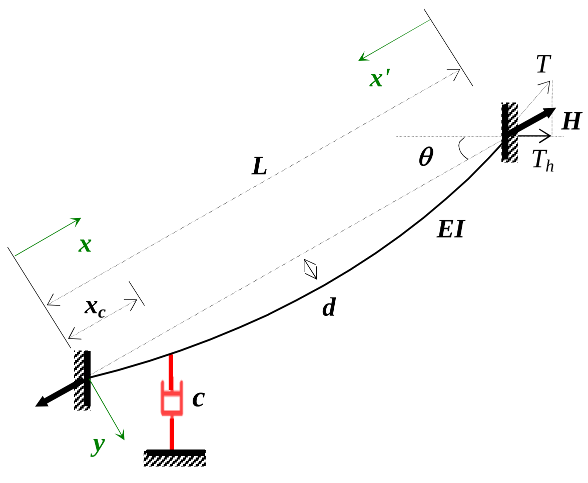

Fig 1. A model of an inclined cable with a damper

For anti-symmetric modes (n = 2, 4, 6...), $R_s = 1$ and $\eta_n = \beta_n^0\, x_c\, c/\sqrt{Hm}$ where $\beta_n^0 = n\pi/L$ is the wave number of a taut cable

3. INFLUENCE OF ANCHOR TUBE STIFFNESS

For an internal damper, i.e., the damper is concealed in the protective tube near the anchorage of the cable, the anchor tube stiffness may have some influence on the performance of the damper. This influence is here considered by a stiffness of the

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Contact address: Department of Civil Engineering, The University of Tokyo, 7-3-1 Hongo Bunkyo-ku, Tokyo 113-8656, Japan
Phone: +81-3-5841-6099; Fax: +81-3-5841-7404

anchor tube, denoted as k, is in tandem with the damping coefficient c (Fig. 2). The damping force is given as $f_c(t) = k\,v_k(t) = c\,[\dot{v}(x_c,t) - \dot{v}_k(t)]$ where $v_k(t)$ is the displacement of the anchor tube at the damper location, from which: $F_c = i\omega c \tilde{v}_c \dfrac{1 - i\omega c/k}{1+(\omega c/k)^2}$. Substituting F_c into Eq. (2):

$$\frac{\xi_n}{x_c/L} = R_s \frac{\eta_n \eta_k^2 (1+\eta_k^2)}{(1+\eta_n\eta_k+\eta_k^2)^2 + \eta_n^2\eta_k^4} \qquad (4)$$

where η_n is as before and $\eta_k = k/\beta_n^{s0} c\sqrt{H/m}$. It is useful to interpret the influence of the tube stiffness as a reduction in this maximum damping value ξ_n^{max} by Eq. (3). The parameter η_k in this condition becomes $\bar{\eta}_k = x_c k/H$ and the resultant damping ratio is ξ_k^{max}. A reduction factor R_k then can be introduced as

$$R_k = \frac{\xi_k^{max}}{\xi_n^{max}} = \frac{2\bar{\eta}_k^2(1+\bar{\eta}_k^2)}{(1+\bar{\eta}_k+\bar{\eta}_k^2)^2+\bar{\eta}_k^4} \qquad (5)$$

The factor R_k indicates the influence of anchor tube stiffness on the maximum damping ratio of the cable with damper and is illustrated in Fig. 2. It is seen that the anchor tube stiffness reduces the damping effect, but not significantly. In addition, since $\bar{\eta}_k$ is independent to the mode index n, the same reduction is applied for any vibration mode to be controlled.

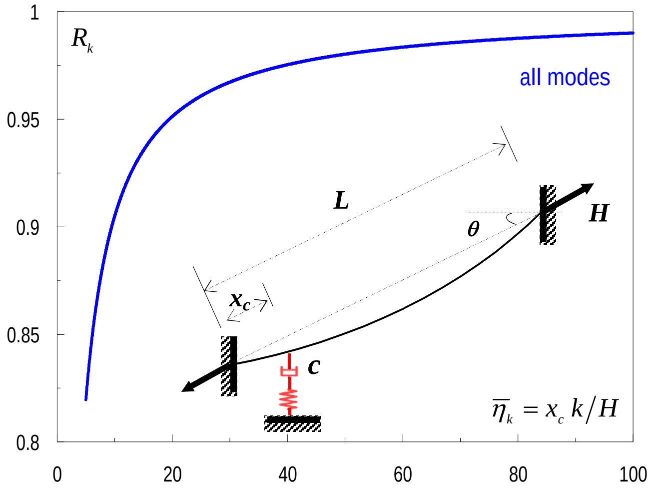

Fig 2. Influence of anchor tube stiffness

4. HIGH-DAMPING-RUBBER DAMPER

For a HDR damper, the damping force is independent of frequency and expressed as $f_c(t) = K(1+i\varphi)\,v(x_R,t)$, from which $F_c = K(1+i\varphi)\,\tilde{v}(x_R)$, where K is the spring factor of the damper; and φ is the loss factor of material. The distance of HDR damper from the cable anchorage is denoted as x_R. Substituting F_c into Eq. (2) give the result

$$\frac{\xi_R}{x_R/L} = \frac{\varphi\eta_R}{(1+\eta_R)^2+(\varphi\eta_R)^2} \qquad (6)$$

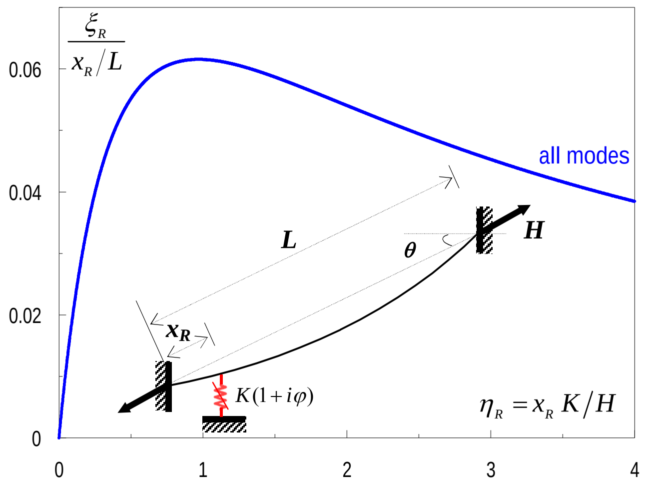

Fig. 3. Estimate curve for ξ_R (φ = 0.25)

where $\eta_R = x_R K/H$ is a non-dimensional parameter of the spring factor. A typical curve showing the variation of ξ_R versus η_R for φ = 0.25 is shown in Fig. 3. A useful feature of using HDR damper can be seen from Eq. (6): ξ_R is independent of mode index. This means the same maximum level of cable damping can be achieved for all vibration modes.

5. COMBINED EFFECT OF TWO DAMPERS

In order to increase the damping effect, using two combined units of dampers at two ends of the cable may be a possible solution. In principle with such a damper configuration further stiffens the cable and the damping performance thus could be superimposed. Similar to the development in case of a single damper, the effect of two dampers on the damping of cable can be also evaluated. The asymptotic expression of modal damping ratio, with $\bar{x} = x_R'/x_c$, is

$$\frac{\xi_n}{x_c/L} \simeq \frac{[\eta_n+\varphi\bar{x}\eta_R+\eta_n\eta_R(1+\bar{x})][1+\eta_R-\varphi\eta_n\eta_R]+[\varphi\eta_n\eta_R(1+\bar{x})-\bar{x}\eta_R][\eta_n+\varphi\eta_R+\eta_n\eta_R]}{[1-\varphi\eta_n\eta_R+\eta_R]^2+[\eta_n+\varphi\eta_R+\eta_n\eta_R]^2} \qquad (7)$$

6. CONCLUSION

Simple formulas of the modal damping ratio of the cables with attached dampers have been analytically derived considering the sag in associated with inclination and stiffness of anchor tube. High-damping rubber devices is also analyzed. Finally the combination of two dampers, each near one end of the cable, is shown to proportionally raise the modal damping level.

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