CENTROIDAL VORONOI SHAPED APPLIED ELEMENT METHOD (CVAEM)

Kawin Worakanchana, Student Member, The University of Tokyo
Kimiro Meguro, Regular Member, The University of Tokyo

1. INTRODUCTION

Applied Element Method[1)] (AEM) is a discrete approach used for collapse analysis. The advantage of AEM is that it can simulate structural behavior from an elastic range to total collapse with reliable accuracy within reasonable CPU time. In this paper, the authors aim to introduce CVAEM which is new version of AEM. The CVAEM contains Voronoi shape element unlike the original version which uses only the square shape. CVAEM advantages are that 1) it is easier fit to any physical domain 2) it reduces the crack directional biased 3) it is able to have a predefined joint face within the domain in any direction and 4) it contains implicit Poisson's ratio (ν). With these additional advantages over the previous version, CVAEM can extend much wider original AEM application.

2. ELEMENT FORMULATION

Element formulation is in the same way as Hexagonal AEM (HAEM[2)]) written by the same author. Due to limitation of space, please refer to the stated reference.

○ Voronoi nuclei ● Centroid

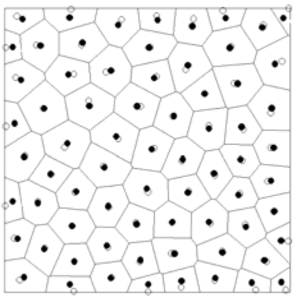

Fig. 1 Voronoi diagram σ=3.76x10^{-1}

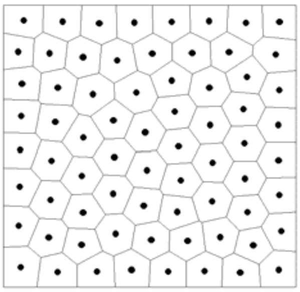

Fig. 2 Voronoi diagram σ=1.74x10^{-7}

3. MESH GENERATION

Mesh generation usually has the significant effect on the elastic properties of discrete element mesh. For example, it is known that a discrete element method with a structured mesh such as hexagonal or rectangular shape shows totally different Poisson's ratio (hexagonal mesh has and rectangular does not have). Unlike a structured mesh, an unstructured basically cannot exhibits the elastic uniformity unless the proper treatment is employed. Schlangen and Garboczi[3)] proposes the iterative approach for assigning beam cross-sectional areas and moment of inertia to obtain the elastic uniformity but the method is complicated and non-unique solution is obtained. Bolander[4)] shows that RBSN, the discrete approach with Voronoi mesh, can simulate the result which is nearly resemblance to elastic uniformity if the element nodes are placed at the Voronoi nuclei and the property of connecting springs vary with element sides. With the success of the unstructured mesh to exhibit good elastic behavior, there is the motivation to apply the unstructured mesh to original AEM. It is noted that although center of mass and element node in RBSN are close to each other, it is not at the same position. The difference in the position is also increased at the boundary element (figure 1). This causes the difficulty in dynamic analysis which inertia force and gravity force becomes significant. The Centroidal Voronoi diagram (figure 2) has the properties that the element centroid is located exactly at the same place as the Voronoi nuclei so the stated problem can be eliminated. Lloyd's method is used in this study to create the Centroidal Voronoi mesh. Because making a perfect Centroidal Voronoi mesh is time-consuming, some error in the difference between the Voronoi nucleus and center of mass position is accepted after some cycle of Lloyd's algorithm. In this study, σ, used as indicator for the mesh quality, is the sum of square of distance between the Voronoi nucleus and centroid as shown in Equation 1 where x and z are vector indicating the position of Voronoi nucleus and centroid, respectively

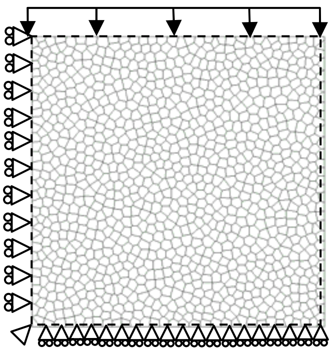

Fig. 3 Example 1 (undeformed)

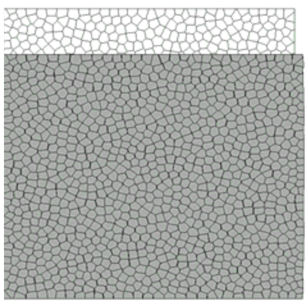

Fig. 4 Deformed mesh

$$\sigma = \sum_{i=1}^{n} \left\| x_i - z_i \right\|^2 \qquad (1)$$

4. ELASTIC BEHAVIOR

The first example shows the behavior of square box under the uniform compressive force in plain stress case (figure 3). Several VAEM meshes were tested to investigate the parameters affecting on the elastic behavior. The first letter in the case name indicates the element size where S is small; M is medium and L is large, respectively. The second number indicates the mesh number and the last number indicate the Lloyd's cycle used in generate the mesh. Mesh 1, 2 and 3 were generated using the method proposed by Bolander[4)]. Mesh 4 is randomly created without any constraint. From figure 7, mesh generation except from the M41 have slightly effect on the elastic Poisson's ratio. The plot between σ and r (ratio between apparent to input ν) shows that for the lower σ, r is approaching 1 for all mesh indicating the better elastic behavior. The comparison between input and apparent ν and Young's modulus (E) shows a good matching for ν between 0 to 0.2 as shown in figure 4 however the error was

Key words: Applied Element Method, Discrete Element, Elastic Analysis
Institute of Industrial Science, the University of Tokyo, 4-6-1 Komaba, Meguro-Ku, Tokyo 153-8505, Japan
(Tel: 03-5452-6437, Fax: 03-5452-6438)

found when ν is approaching 0.3 for both ν and E. Also, figure 5 shows that the input ν starts affecting apparent E when it reaches to 0.3. The result of varying E was shown in figure 6 at $\nu = 0$. The good matching between theoretical and numerical value is obtained.

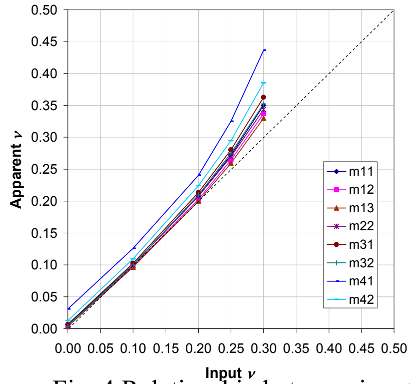

Fig. 4 Relationship between input and apparent ν

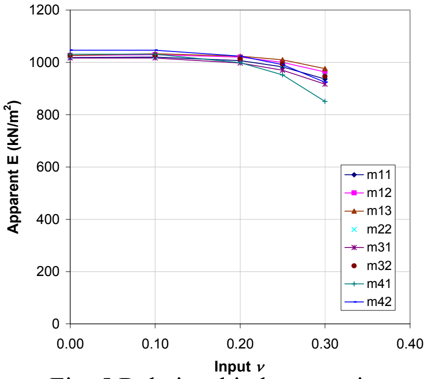

Fig. 5 Relationship between input ν and apparent E

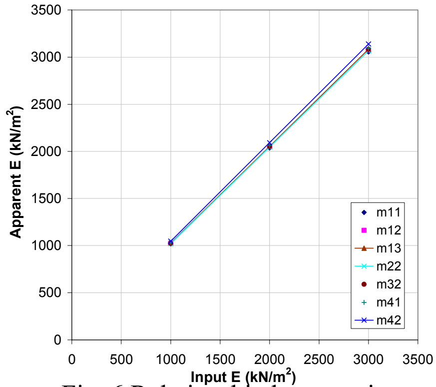

Fig. 6 Relationship between input and apparent E

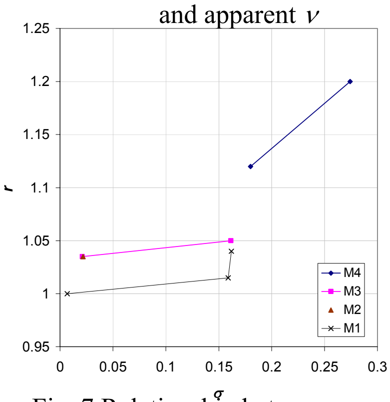

Fig. 7 Relationship between σ and ratio of apparent ν to real ν (r)

5. FRACTURE BEHAVIOR

Fracture in AEM is represented by failure of a spring connecting between 2 elements. In CVAEM, the failure criterion is described as the followings:

$$\left(\frac{\sigma_n^2+\sigma_s^2}{\sigma_t^2}\right)\geq 1, \sigma_t \geq 0 \qquad (2)$$

where σ_n and σ_s are normal and shear stress and σ_t are tensile strength. The equation means the spring will break when stress in the direction of total force reaching the tensile strength only if the normal spring is in tension. To verify the cracking criteria, CVAEM is used to simulated double-edge-notched (DEN) specimen[5] (figure 8). The test specimen is shown in figure 10. The case simulated here following the load path 1 in the experiment where the pre-shear force is 10 kN. The good matching between experimental crack distribution and the simulation was observed as shown in figure 8. The P-δ obtained from the numerical model exhibits the faster drop in resisting force after the peak load (figure 9). This is due to the model assumption that the force in the spring drop to 0 suddenly after breaking. This can be improved later by changing the stress-strain relation of concrete after a peak.

6. CONCLUSION

The CVAEM was developed in this paper. The elastic behavior is well matched in the range of ν from 0 to 0.2. The model also shows good capability to follow the crack pattern.

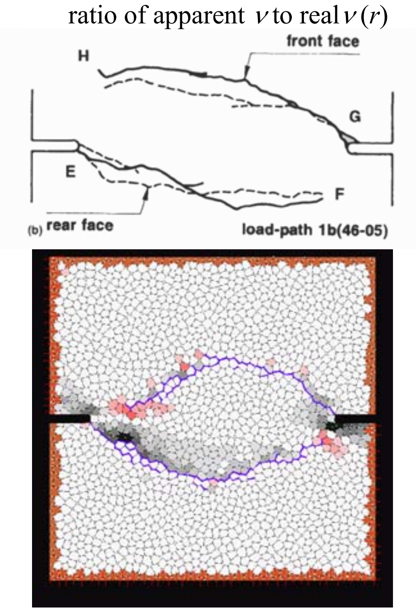

Fig. 8 Comparison of simulated and experimental crack pattern

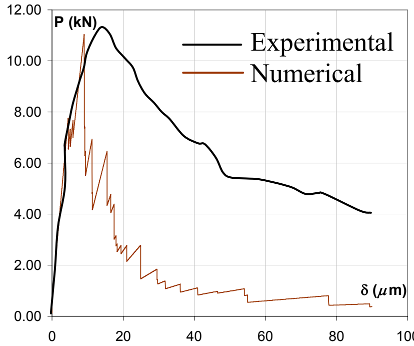

Fig. 9 Comparison between experimental and numerical P-δ

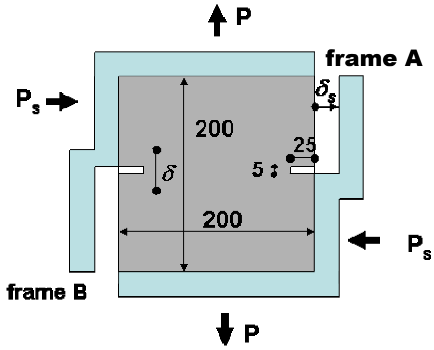

Fig. 10 DEN test setup

7. REFERENCE

1. Meguro, K., and Tagel-Din, H., 2001. Applied Element Simulation of RC Structures under Cyclic Loading. *Journal of Structural Engineering*, ASCE. 127, 11: 1295-1305.
2. Worakanchana, K., and Meguro K., 2004. 2-D Elastic Analysis Using the Hexagonal Shape Applied Element Method, Paper No. 1-352 - 9, 2004.
3. Schlangen, E. and Garboczi, E. J., 1996 New Method for Simulating Fracture using an Elastically Uniform Random Geometry Lattice. *International Journal of Engineering Science*, Pergamon. 34, 10: 1131-1144.
4. Bolander, J.E., and Saito, S., 1998. Fracture analyses using spring networks with random geometry. *Engineering Fracture Mechanics*, Pergamon. 61: 569-591.
5. Nooru-Mohamed, M.B., Schlangen, E. and Van Mier, G.M., 1993. Experimental and Numerical study on the Behavior of Concrete Subjected to Biaxial Tension and Shear. *Advanced Cement Based Materials*, Elsevier. 1,1: 22-3