PRELIMINARY INVESTIGATION FOR EXTENSION OF FEM-β TO ELASTO-PLASTICITY

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1. Introduction

Current available methods for structural analysis cannot follow the failure behavior accurately. Widely used Finite Element Method (FEM) solves the boundary value problem (BVP) precisely. However, it cannot express displacement discontinuity as it uses smooth overlapping shape functions. On the other hand, Distinct Element Method (DEM) can follow the failure behavior. However, the spring constants can not be rigorously determined. FEM-β[1)], a new method for numerical simulation for failure behavior of continuum has been developed to cover the above mentioned problems. FEM-β provides a rigorous block-spring model for deformable body with easy treatment of failure. Here, an extension of FEM-β to perform elasto-plastic analysis is achieved.

2. FEM-β

The particle discretization is applied to a BVP of two dimensional homogeneous elastic body V. When the displacement $\bar{u}_i$ is prescribed on the boundary ∂V, The BVP for displacement u_i is posed as

$$c_{ijkl}u_{k,li} = 0 \quad \text{in } V, \qquad u_i = \bar{u}_i \quad \text{on } \partial V. \tag{1}$$

where c_{ijkl} is the elasticity tensor.

The domain is decomposed into a set of Voronoi blocks. The non-overlapping characteristic functions[1)] of the Voronoi blocks, see **Fig. 1**, and the Delaunay triangles, respectively, are used to discretize the displacement and stress functions as follows

$$u_i(\mathbf{x}) = \sum_{\alpha=1}^{N} u_i^\alpha \phi^\alpha(\mathbf{x}) \quad , \quad \sigma_{ij}(\mathbf{x}) = \sum_{\beta=1}^{K} \sigma_{ij}^\beta \gamma^\beta(\mathbf{x}) \tag{2}$$

$$\phi^\alpha(\mathbf{x}) = \begin{cases} 1 & \text{for } \mathbf{x} \in \Omega^\alpha \\ 0 & \text{for } \mathbf{x} \notin \Omega^\alpha \end{cases} \quad , \quad \gamma^\beta(\mathbf{x}) = \begin{cases} 1 & \text{for } \mathbf{x} \in \Delta^\beta \\ 0 & \text{for } \mathbf{x} \notin \Delta^\beta \end{cases} \tag{3}$$

where: ϕ^α is the characteristic function on α^{th} Voronoi block Ω^α and γ^β is the characteristic function on β^{th} Delaunay triangle Δ^β.

The strain cannot be computed from the discretized displacement in Eq. (2) since the derivative of ϕ^α becomes a delta function along the block boundaries $\partial\Omega^\alpha$. FEM-β computes the average strain to evaluate the stress over the triangular domain. The contribution of the displacement $\mathbf{u}^1$ of the point $\mathbf{x}^1$ to the average strain over Δ^{123}, **Fig. 2**, can be computed[1)] as

$$\begin{bmatrix} \bar{\varepsilon}_{11} \\ \bar{\varepsilon}_{22} \\ 2\bar{\varepsilon}_{12} \end{bmatrix} = \frac{1}{2A}\begin{bmatrix} x_2^2 - x_2^3 & 0 \\ 0 & x_1^2 - x_1^3 \\ x_1^2 - x_1^3 & x_2^2 - x_2^3 \end{bmatrix}\begin{bmatrix} u_1^1 \\ u_2^1 \end{bmatrix} = \begin{bmatrix} B^1 \end{bmatrix}_{3x2}\begin{bmatrix} u^1 \end{bmatrix}_{2x1} \tag{4}$$

Similarly, the contributions of $\mathbf{u}^2$ and $\mathbf{u}^3$ are computed and a complete matrix $[B]_{3x6}$ is obtained. Then the average stress is computed as $[c][\bar{\varepsilon}]$ over Δ^{123} where $[c]$ is the elasticity matrix. Eq. (4) shows that the average strain of FEM-β is equal to the uniform strain of ordinary FEM with triangular elements. Hence, both methods produce the same stiffness matrix. This means that FEM-β has the same accuracy of ordinary FEM in computing the displacements, strains and stresses while the displacement field is discretized with non-overlapping characteristics functions.

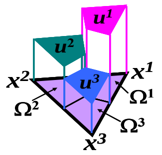

Fig. 1 Pictorial view of the characteristic functions

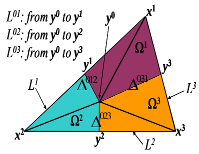

Fig. 2 Triangle domain Δ^{123} consists of parts of blocks $\{\Omega^1,\Omega^2,\Omega^3\}$

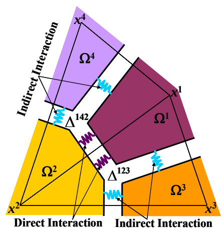

Fig. 3 Six springs connect Ω^1 and Ω^2

Straightforward extension of FEM-β to solve problems with material nonlinearity is achieved. Steel plasticity behavior is considered. Von Mises, the most popular yield criterion for steel, is employed to determine the stress level at which the plastic deformation begins. As the mild steel exhibits plastic flow under constant stress, a simple elasto-prefect plastic stress-strain relationship is adopted. After computing the average strain $\bar{\varepsilon}_{ij}$, the stress calculation in every triangular domain is done via Backward-Euler algorithm, the most accurate return mapping algorithm[2)]. Stress is computed[3)] as $\boldsymbol{\sigma}_\mathbf{F} = \boldsymbol{\sigma}_\mathbf{S} + \mathbf{c}(\Delta\bar{\boldsymbol{\varepsilon}} - \Delta\bar{\boldsymbol{\varepsilon}}_\mathbf{p})$ over Δ^{123} where $\boldsymbol{\sigma}_\mathbf{S}$ is the starting stress, $\boldsymbol{\sigma}_\mathbf{F}$ is the final stress on the yield surface, $\Delta\bar{\boldsymbol{\varepsilon}}_\mathbf{p}$ is the average plastic strain and $\mathbf{c}$ is the elasticity matrix. In this sense, the plasticity is not localized within the triangular domain. To perform more realistic elasto-plastic analysis, a better algorithm is required. The expected algorithm should allow some parts of the triangular domain to go to the plastic zone while the others remain in the elastic zone.

3. Failure Analysis:

The main advantage of FEM-β is the high efficiency in solving failure problems. Using FEM-β, the particle discretization scheme that uses non-overlapping characteristic functions provides a rigid-body-spring model which is equivalent to continuum model. The global stiffness matrix represents springs which connect the rigid bodies, see **Fig. 3**. The matrix components, which are rigorously determined in terms of material properties, give the springs' constants. In every triangular domain, the material strength is used to check failure of springs. The springs to be broken can be picked up[1)] according the direction of the major principal stress. The pervious treatment is not justified yet. A more realistic

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treatment is still needed.

The procedure of failure is described in our program as follows. For all triangular domains, the major principal stress is checked during each iteration of each load increment in the nonlinear solution. If the major principal stress in any triangular domain reaches the given material strength, the corresponding springs are broken. The contributions of the broken springs to the components of the global stiffness matrix are reduced to zero. These reductions generate relatively high residual forces in additional to the residual forces of the nonlinear iterations. All these forces must be redistributed in the next iteration in a similar way to the conventional nonlinear solution. The iterations continue until the residual forces converge to a given tolerance. Then the next load increment is applied. It should be emphasized that the load increments should be small enough to follow the failure behavior. The program checks the global failure after every spring failure. The program stops if the global failure is achieved. If not, the iterations process continues and the next load increments are applied. This preliminary procedure enables some springs to go to the plastic region and some to reach the failure status while the others remain in the elastic region. Moreover, the spring is broken when it just reaches the failure status. Hence, no springs sit beyond the failure criterion. However, the spring failure is checked inside the iteration loop where the solution is not converged yet. Further development is still needed to ensure the breakage of the spring after the convergence of the nonlinear solution. The spring should be have the option to reconnected again during the iterations.

4. Failure Simulation

Failure analysis of an isotropic elasto-plastic steel plate under uniaxial tension in the vertical direction is examined. The plate is square (2x2 m) and it has a circular hole (Diameter=0.2 m) in its center. A plane strain condition is assumed. Young's modulus, Poisson ratio and yield stress are set as E=2100 t/cm^2, υ=0.3 and σ_y=2.4 t/cm^2. The maximum tensile stress is given as 3.6 t/cm^2 to represent the material strength. A total 0.20% uniform elongation is applied in fine increments. A tolerance of 10^{-4}% is given to check the convergence of the nonlinear solution. Snapshots of the distribution of the equivalent Von Mises stress are shown in **Fig. 4a**. White color indicates the plastic triangular domains while black color indicates the elastic and unloaded triangular domains. As expected, some triangular domains go to the plastic region with increase of loading. Springs are broken when they reach the failure status. Breakage of springs causes some triangular domains to unload to the elastic region as a new free surface is being created. Snapshots of the failure pattern are shown in **Fig. 4b**. As the load increases, some springs are broken. This process continues until the global failure is achieved and the plate is broken into two parts.

Conclusions

In this paper, we present the preliminary investigation for extension of FEM-β to perform elasto-plastic analysis of steel structures. Further development to localize the plasticity within the triangular domain is still required. A failure procedure is proposed. Improvement to allow reconnection of the springs during the iterations should be developed. A failure simulation of an elasto-plastic steel plate under uniaxial tension is presented. The results show that FEM-β can handle failure initiation and propagation until a global failure is achieved.

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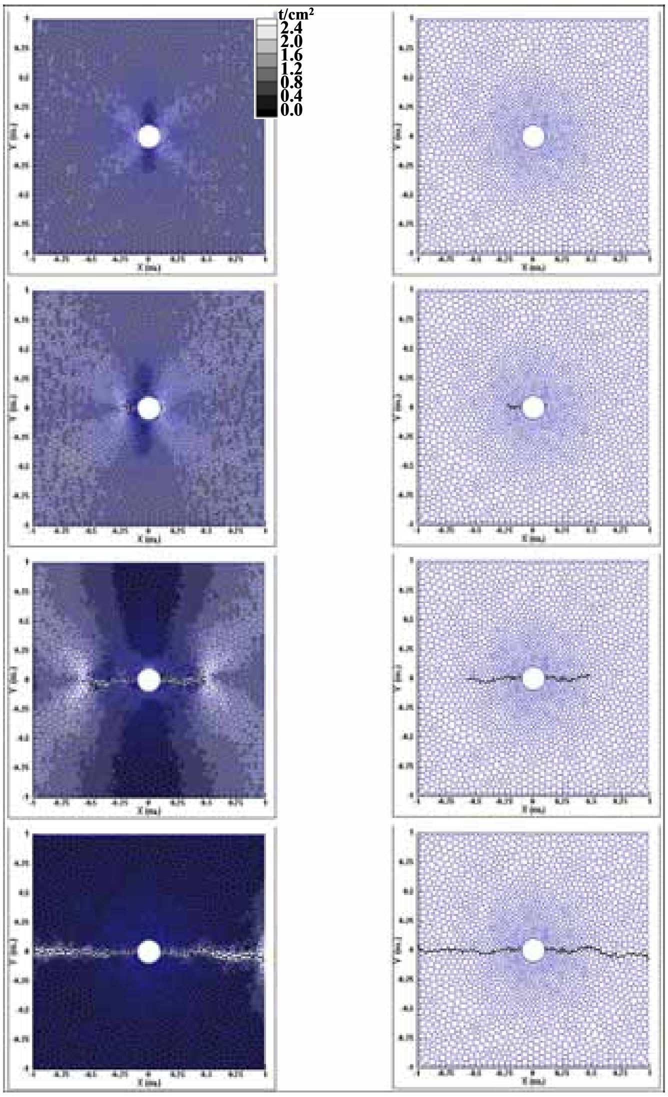

Fig. 4a Equivalent Von Mises stress distribution **Fig. 4b** Failure pattern of an elasto-plastic steel plate under uniaxial tension in the vertical direction