Fluid Flow-Induced Vibrations of Cylindrical Shells on Elastic Foundations

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1. Introduction

Cylindrical shells are widely used in many engineering fields for conveying liquid. Some of them are laid on the foundation. Free vibrations of cylindrical shells with a non-axisymmetric elastic bed have been investigated by Amabili et al.[1] based on the Rayleigh-Ritz method. Complicating effects due to contained invicid liquid, intermediate constraints, and added mass are considered later on by Amabili et al.[2]. Lakis and Sinno [3] have developed the combined formulation of finite element and classical shell theory which is applied for free vibration of empty and liquid filled shells. This paper presents the free vibration characteristics of cylindrical shells conveying fluid on elastic foundations. Similar method as described by Gunawan et al.[4] is adopted. Numerical results are presented to give better pictures of the dynamic characteristics of the problem.

2. Model and Formulation

The shell considered is an isotropic thin elastic shell based on the Sanders theory with Young's modulus E, Poisson's ratio υ, mass density ρ_S, radius of the middle surface R, thickness h, and length L. The foundation may be represented by continuous elastic (axial, circumferential, radial, and rotational) springs on a limited arc which corresponds to an angle $\varphi_1+\varphi_2$. Geometry of the structure and generalized model with reference direction are shown in Fig.1, where K_u, K_v, K_w, and K_β denote the associated directional spring coefficients.

The displacements of a point on the middle surface in axial, circumferential, and radial direction are indicated by u, v, and w, respectively and are expressed as follows:

$$\begin{aligned}
u(x,\theta) &= \sum_{m=0}^{M}\left\{U_m^{\mathrm{S}}(x)\cos(m\theta)+U_m^{\mathrm{U}}(x)\sin(m\theta)\right\}\\
v(x,\theta) &= \sum_{m=0}^{M}\left\{V_m^{\mathrm{S}}(x)\sin(m\theta)+V_m^{\mathrm{U}}(x)\cos(m\theta)\right\}\\
w(x,\theta) &= \sum_{m=0}^{M}\left\{W_m^{\mathrm{S}}(x)\cos(m\theta)+W_m^{\mathrm{U}}(x)\sin(m\theta)\right\}
\end{aligned} \tag{1}$$

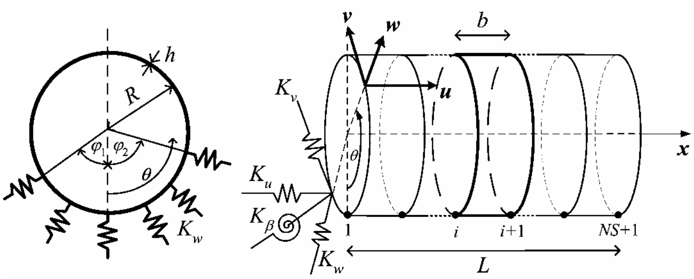

Fig. 1. Geometry and discretization of the problem.

Superscripts S and U refer to symmetrical and asymmetrical systems. Typical circumferential wave is denoted by m in Eq.1. The shell is filled fully with the fluid and the fluid is assumed to be invicid, incompressible, and irrotational so that the flow can be described by a velocity potential Φ which satisfies the Laplace equation. The hydrodynamic pressure acting on the wall of the shell is determined from the linearized Bernoulli equation and is given as follows: $p=-\rho_{\mathrm{L}}(\partial\Phi/\partial t+U\,\partial\Phi/\partial x)$, where ρ_{L} and U are mass density and axial velocity of the fluid, respectively. The motion of the shell and fluid is coupled by the radial velocities on the interface between shell and fluid. The velocity potential function may be expressed as a summation of the following function $\Phi_{mj}=\Phi_{mj}^{\mathrm{S}}+\Phi_{mj}^{\mathrm{U}}$ over the whole ranges of m and j. By taking into account the mentioned assumptions, one may found the explicit form of Φ as a function of w.

By minimization of the total potential energy of the vibrating system and using the finite element method, the following governing equation can be obtained:

$$\left[\mathbf{M}_{\mathrm{S}}+\mathbf{M}_{\mathrm{L}}\right]\left\{\ddot{\mathbf{d}}\right\}+\left[\mathbf{C}_{\mathrm{L}}\right]\left\{\dot{\mathbf{d}}\right\}+\left[\mathbf{K}_{\mathrm{S}}+\mathbf{K}_{\mathrm{F}}+\mathbf{K}_{\mathrm{L}}\right]\left\{\mathbf{d}\right\}=0 \tag{2}$$

where $\mathbf{M}_\mathrm{S}$ and $\mathbf{K}_\mathrm{S}$ are mass and stiffness matrices of the shell; $\mathbf{M}_\mathrm{L}$, $\mathbf{C}_\mathrm{L}$, and $\mathbf{K}_\mathrm{L}$ are mass, damping, and stiffness matrices of the flowing fluid; and $\mathbf{K}_\mathrm{F}$ is the stiffness matrix of the foundation. The first, second, and third terms in Eq.2 are denoted by **M**, **C**, and **K**, respectively. Eq.2 is a quadratic eigenvalue problem and can be solved by a linearization process. Note that, this linearization is not unique.

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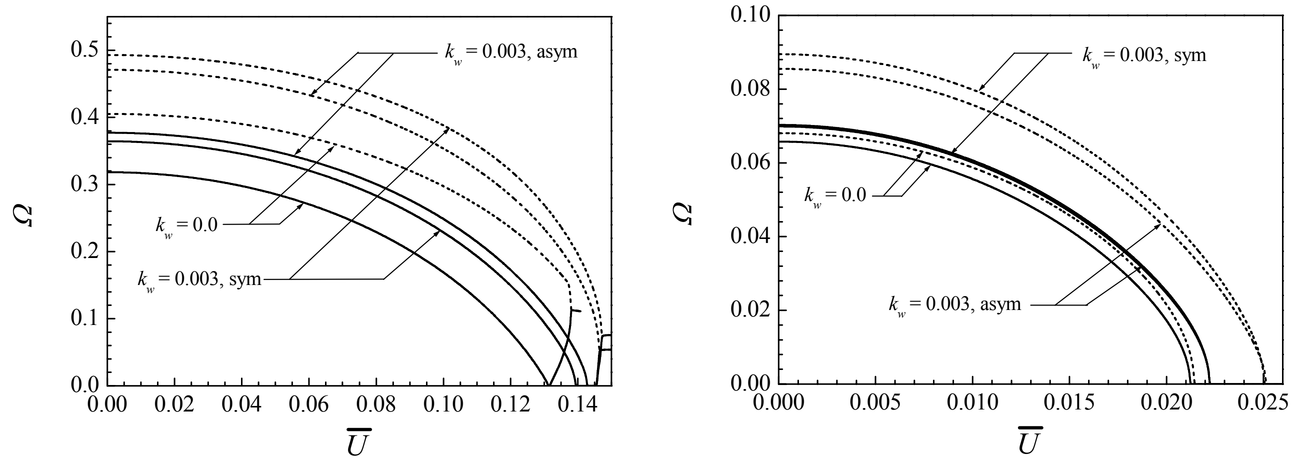
$$\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \begin{Bmatrix} \mathbf{d} \\ \dot{\mathbf{d}} \end{Bmatrix} = \Lambda \begin{Bmatrix} \mathbf{d} \\ \dot{\mathbf{d}} \end{Bmatrix} \tag{3}$$

where Λ = iω. For convenient, a non-dimensionalized frequency parameter $\Omega = \omega L\sqrt{\rho_S(1-\upsilon^2)/E}$ is used.

3. Numerical Results

Based on the numerical investigation which is not shown here, convergence of the solution is affected by the flow velocity. For very high velocity flow, more elements are needed to assure the accuracy. Therefore, total number of elements NS = 40 and M = 20 are used through out this study. Numerical examples are presented for simply supported shell at both ends with $\varphi_1 = \varphi_2 = \varphi$. In the analysis, only radial spring is considered ($K_u = K_v = K_\beta = 0$). Fig.2 shows the results.



(a) R/L = 0.10, R/h = 20 (b) R/L = 0.30, R/h = 200

Fig. 2. Variations in Ω of the first mode with non-dimensionalized velocity $\overline{U} = U\sqrt{\rho_S(1-\upsilon^2)/E}$.
(Simply supported shell, υ = 0.30, K_wL/E = 0.003, $\varphi = \pi/3$, and ρ_S/ρ_L = 0.128)

4. Conclusions

Fluid flow-induced vibrations of cylindrical shells on elastic foundations are shown by the semi-analytical hybrid finite element method. The method inherits the versatility of the Finite Element Method in applying the boundary conditions. Non-uniformities of the foundations both in the circumferential and longitudinal directions can be simply taken into consideration by means of the Fourier series and an element mesh strategy. Convergence is affected by the flow velocity. Generally, more elements are needed for high flow velocity to assure the accuracy of the solution.

Natural frequencies of the vibrating system are decreased by the flow of the fluid. At a certain velocity the system lost its stability due to divergence. Both shells with or without foundation behave similarly. However, the existence of the foundation increases the natural frequency and the critical velocity. It has been observed that the effect of the foundation is more pronounced for shells with small values of R/L and R/h.

References:

1) Amabili, M. and Dalpiaz, G. Free Vibrations of Cylindrical Shells with Non-axisymmetric Mass Distribution on Elastic Bed. Meccanica 1997; 32:71-84.
2) Amabili, M. and Garziera, R. Vibrations of circular cylindrical shells with non-uniform constraints, elastic bed and added mass; Part I: Empty and fluid-filled shells, Journal of Fluids and Structures 2000; 14(5): 669-690
3) Lakis, A.A. and Sinno, M. Free Vibration of Axisymmetric and Beam-like Cylindrical Shells, Partially filled with Liquid. International Journal for Numerical Methods in Engineering 1992; 33:235-268.
4) Gunawan, H., Sato, M., Mikami, T., Kanie, S. Static and free vibration analysis of cylindrical shells on elastic foundation. Journal of Structural Engineering JSCE 2004; 50A:25-34.
5) Gunawan, H., Mikami, T., Kanie, S., Sato, M. Application of Shell Equations Based Shape Function for Free Vibration Analysis of Cylindrical Shells on Elastic Foundation. Proceedings of 59 th JSCE Conference 2004:675-676.