Performance of automatic load stepping method for FEM with subloading tij model

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ABSTRACT

In this research the performance of an automatic load stepping method proposed by Abbo and Sloan, in the displacement non-linear finite element method, is investigated for the elastoplastic subloading t_{ij} model. The step size in this method is controlled with the local truncation error which is measured by computing the difference between two estimates of the displacement increments of each load steps. If the local truncation error exceeds a specified tolerance, then the load step is abandoned and a new size of the load step is adapted by extrapolation. It is found that the scheme successfully controls the load path error regardless of the size of the load increment.

1. INTRODUCTION

In finite element method analyzing non-linear material, the load is applied as a number of increments. The size of the load increment has a vital influence on the convergence and accuracy of a solution. Analysis of critical state model with finite load increments inevitably causes some load path error in the resulting displacement and stresses. This error increases with bigger increment of load. Bigger increment size produces inaccurate results in a highly non-linear problem. To get an acceptable accuracy it is required to employ the appropriate size of the increment, which can only be found by trial and error. Sometimes it is difficult to determine appropriate size of the loading increment to converse the solution of the analysis. Abbo and Sloan [1] proposed an automatic loading scheme for explicit integration which automatically adjusts the increment size when computing the load-displacement behavior. The automatic load increment scheme controls the load path error choosing appropriate load increment automatically. The method is robust and efficient for conventional plasticity models. Subloading tij model [2], which can describe properly the influence of intermediate principal stress on the deformation and strength of soils, dependence of the direction of plastic flow on the stress paths, influence of density and/or confining pressure on the deformation and strength of soils, is used as a constitutive model of soils.

2. AUTOMATIC LOAD INCREMENT SCHEME

In the displacement finite element method adopting the first order forward Euler load stepping method, the equations of global stiffness is solved for each load increment as

$$\Delta u = [K(u_0)]^{-1} \Delta F^{ext} \tag{1}$$

$$u = u_0 + \Delta u \tag{2}$$

Where, Δu is the displacement increment vector, u_0 is the displacement vector at the start of the load increment, u is the displacement vector at the end of the load increment, $[K]$ is the stiffness vector and ΔF^{ext} is a vector of external force increments.

In the automatic load increment scheme, equation (1) is solved employing a pseudo time substep in the range of $0 < \Delta T \le 1$ and a subincremental load vector of $\Delta F_a^{ext} = \Delta T \Delta F^{ext}$. The automatic load increment scheme algorithm consists of the following steps (here the unbalanced force is not included):

1. Current stresses σ, current displacements u, external load increment and previous pseudo time substep ΔT_{last} and displacement error tolerance $DTOL$.
2. Initialize: $T = 0$; $\Delta T = \min\{\Delta T_{last}, 1\}$
3. While T>1 go to step 10.
4. Compute: $\Delta F_a^{ext} = \Delta T \Delta F^{ext}$; $\Delta u_1 = [K(u)]^{-1} \Delta F_a^{ext}$;
5. Compute first-order displacement update: $u_1 = u + \Delta u_1$

   And then integrate constitutive law to find corresponding state of stress σ_1.
6. Compute: $\Delta u_2 = [K(u_1)]^{-1} \Delta F_a^{ext}$
7. Compute error: $Err = \dfrac{\frac{1}{2}\|\Delta u_2 - \Delta u_1\|}{\|u_n\|}$
8. If $Err \le DTOL$ go to step 9. Else current load subincrement has failed, so estimate a smaller pseudo time step using

   $q = \max\left\{0.7\sqrt{\dfrac{DTOL}{Err}}, 0.1\right\}$, and set $\Delta T = q\Delta T$, and return to step 4.
9. Current load step is successful. Update pseudo time, displacements, and stresses according to

   $T = T + \Delta T$; $u = u_1$; $\sigma = \sigma_1$

   $q = \max\left\{0.7\sqrt{\dfrac{DTOL}{Err}}, 1.1, \dfrac{1-T}{\Delta T}\right\}$, and set $\Delta T = q\Delta T$, and return to step 3.
10. Save size of the last increment: $\Delta T_{last} = \Delta T$

Key words: Automatic load stepping, Finite element method, Elastoplastic model
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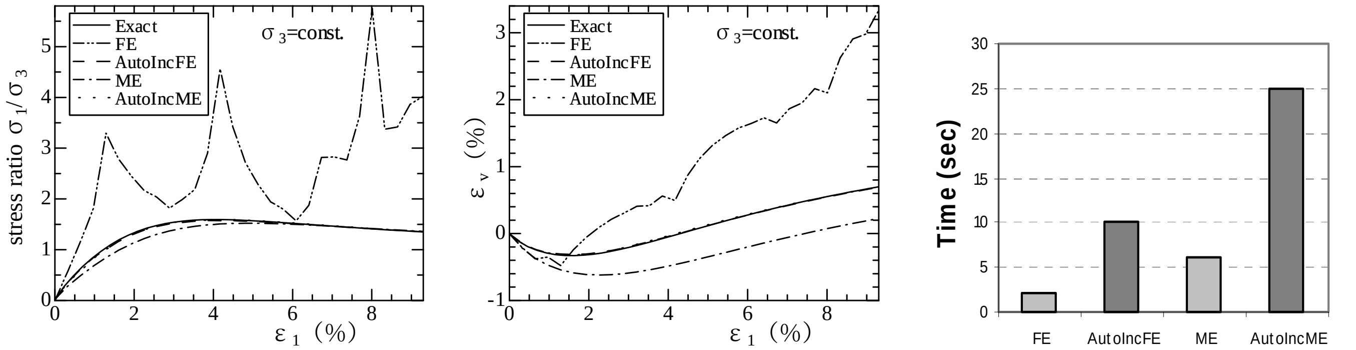
3.1 TRIAXIAL TEST PROBLEM:

The automatic load stepping is used to analyze the behavior of triaxial test for aluminum rod mass. Two integration schemes for integrating the stresses are employed: one is forward Euler (FE) method and the other is modified Euler method (ME) with error control [4]. 30 load increments are applied in this analysis. For the auto sub-load increment scheme *DTOL* equal to 10^{-3} is used. In the modified Euler stress integration scheme the tolerance of 10^{-4} is taken for both the FE load increment and auto load increment steps. Fig. 1 shows the results of the analysis. Here, the results of FE, auto sub-load increment FE, ME and auto sub-increment ME are compared with the results of the exact solution. It is seen in Figs. 1(a) and (b) that the coarse load increment of 30 steps gives inaccurate results in case of the ordinary FE method. Although the ME stress integration scheme gives better solution compare to the FE integration scheme, a significant drift from the equilibrium is observed specially in the dilatancy curve. The auto load increment method with 30 coarse load increments produces the same results as the exact solution for both the FE and ME stress integration schemes. Even the auto load increment method with 10 coarse load increments can simulate the problem with good accuracy (here it is not included because FE method has failed to reach to the end of the calculation). The bar diagram (Fig. 1(c)) shows that the FE stress integration scheme requires less CPU time compare to the ME stress integration scheme because the extra iterations for the convergence of the stresses are not required in case of FE stress integration scheme.

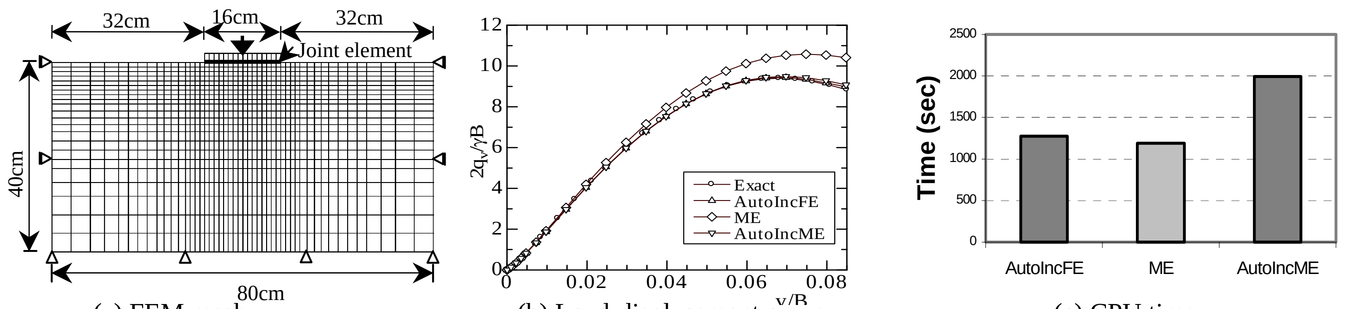


(a) stress-strain curve (b) dilatancy curve (c) CPU time

Fig. 1: Results of triaxial analysis for 30 load increments

3.2 BEARING CAPACITY PROBLEM:

Fig 2(a) shows the mesh for finite element analyses including the geometry of the ground for the bearing capacity problem. The footing is 16cm in width. The mesh consists of 1040 elements and 1112 nodes. Elastoplastic joint elements are used at the interface between the footing and the ground surface. The analyses are carried out for plane strain drained conditions. Both lateral faces of the numerical ground are fixed in horizontal direction, and these are free in vertical direction. The same tolerances and material parameters as triaxial test are used in this analysis. The applied load increment size is 0.004cm. Fig. 2(b) shows the load-displacement curve, and Fig. 2(c) illustrates CPU time for the analyses. For this size of the increment the analysis with FE stress integration scheme without automatic load sub-stepping did not converse the solution and ME scheme over-predicts the bearing capacity. The auto load increment method gives almost the correct solution for both the FE and ME stress integration schemes for this coarse load increment the same as the triaxial test problem.



(a) FEM mesh (b) Load-displacement curve (c) CPU time

Fig. 2: Geometry and results of bearing capacity analysis

4. CONCLUSIONS

The automatic load stepping scheme of Abbo and Sloan can successfully control the load path error regardless the size of the load increment. The method is robust and efficient for the elastoplastic subloading model.

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