ESTIMATION OF PHASE CHARACTERISTICS OF EARTHQUAKE MOTION USING THE CONCEPT OF STOCHASTIC DIFFERENTIAL EQUATION

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1. Introduction

Modeling the non-stationary characteristics of earthquake motion necessitates that the phase characteristic of earthquake motion should be properly modeled to simulated realistic earthquake motion. In this paper, the concept of group delay time which is expressed in linear stochastic differential equation is used to simulate the phase spectra of earthquake motion. Extended Kalman filter was used to identify the unknown parameters of the stochastic differential equation together with the observable characteristics of group delay time (mean and variance). The phase spectra are then generated from the model.

２. Simulation of group delay time using the concept of differential equation

Group delay time is defined as,

$$t_{gr}(\omega)=\frac{d\phi(\omega)}{d\omega} \tag{1}$$

Using real and imaginary parts of Fourier spectrum, Eq. (1) can be expressed by,

$$t_{gr}(\omega)=\frac{F_R(\omega)\cdot\frac{dF_I(\omega)}{d\omega}-F_I(\omega)\cdot\frac{dF_R(\omega)}{d\omega}}{F_R(\omega)^2+F_I(\omega)^2} \tag{2}$$

Expression of group delay time is assumed to be given by a linear stochastic differential equation

$$dt_{gr}(\omega)=\left(c_1(\omega)t_{gr}+c_2(\omega)\right)d\omega+\left(\sigma_1(\omega)t_{gr}+\sigma_2(\omega)\right)dB_\omega \tag{3}$$

where, $c1(\omega)$, $c2(\omega)$, $\sigma_1(\omega)$ and $\sigma_2(\omega)$ are the unknown parameters, $dt_{gr}(\omega)$ and dB_ω are the increment of group delay time and that of Brownian noise at a discritized frequency.

To use the extended Kalman filter for identifying the unknown parameters, we derived the differential equations of first order for the mean group delay time, Eq.(4) and the variance group delay time, Eq.(5) from Eq.(3) as,

$$\frac{d\mu_{tgr}(\omega)}{d\omega}=c_1(\omega)\mu_{tgr}+c_2(\omega) \tag{4}$$

$$\frac{d\sigma_{tgr}(\omega)^2}{d\omega}=\sigma_1(\omega)^2\sigma_{tgr}{}^2+2\left[c_2(\omega)+\sigma_1(\omega)\sigma_2(\omega)\right]\mu_{tgr}+\sigma_2(\omega)^2 \tag{5}$$

３. Numerical Simulation

Smoothing the data of the observed Hyogoken-Nanbu earthquake spectrum

The observed earthquake motions from the Hyogoken-Nanbu earthquake motion with a sampling interval of 0.01(sec) is shown in Figure 1. In order to reduce the abnormal nature of fluctuation of the group delay time, we only took 99.7% confidence interval ($\mu_{tgr}(\omega_i)\pm3\sigma(\omega_i)$) of data for every 5Hz of frequency interval. Both the fluctuating data and treated data of group delay time are shown in Figure 2. The sampling data was also smoothened using weighted moving average. The spectral window, k is 10rad/s for every frequency interval. Figure 3 shows both weighted moving average of mean, $\mu_{tgr}(\omega_i)$and variance, $\sigma_{tgr}{}^2(\omega_i)$ of group delay time.

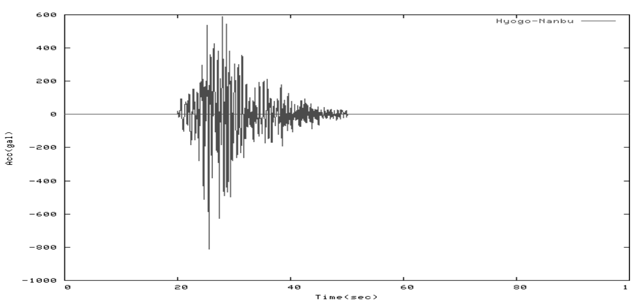

Fig. 1 Hyogo-Nanbu earthquake motion

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Solving the unknown parameters using Extended Kalman Filter

By introducing the extended Kalman filter, the unknown parameters of Eq.(4) and Eq.(5), from the measured data, $\mu_{tgr}(\omega_i)$ and $\sigma^2{}_{tgr}(\omega_i)$ of the observed group delay time in frequency interval of 10 rad/s, $d\omega_i$ can be solved.

Consider a system as state space model described by,

$$\mathbf{x}_{\mathbf{i+1}} = \mathbf{x}_{\mathbf{i}} + \mathbf{w}_{\mathbf{i}} \tag{6}$$

$$\mathbf{y}_{\mathbf{i}} = \mathbf{h}_{\mathbf{i}}(\mathbf{x}_{\mathbf{i}}) + \mathbf{v}_{\mathbf{i}} \tag{7}$$

where $\mathbf{x}_{\mathbf{i}}$ and $y_{\mathbf{i}}$ are the system vector and the measurement, respectively and $\mathbf{h}_{\mathbf{i}}$ is the vector to be estimated. The respective exogenous noises signals $\mathbf{w}_{\mathbf{i}}$ and $\mathbf{v}_{\mathbf{i}}$ are the process and measurement noises.

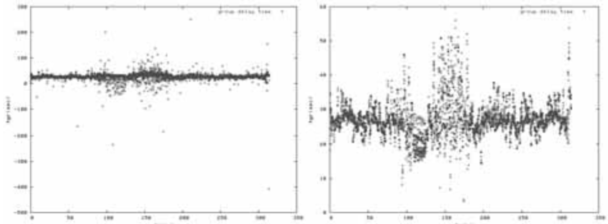

Figure 2: The fluctuating data on the left while 99.7% confidence interval treated data of group delay time on the right

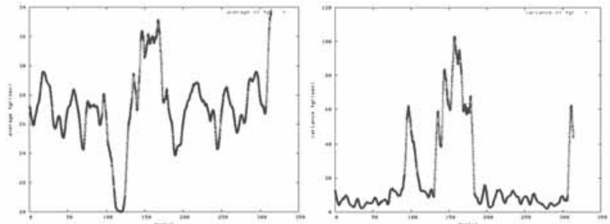

Figure 3: weighted moving average of mean (right) and variance (left) of group delay time.

Observation:

$$y_i = \left[\frac{d\mu_{tgr}}{d\omega}\bigg|_{\omega=\omega_i} \quad \frac{d\sigma^2{}_{tgr}}{d\omega}\bigg|_{\omega=\omega_i} \right]^T \tag{8}$$

Unknown parameters:

$$x_i = \left[c_1(\omega_i), \quad c_2(\omega_i), \quad \sigma_1(\omega_i)^2, \quad \sigma_2(\omega_i)^2 \right]^T \tag{9}$$

And observation functions:

$$\mathbf{hi} = \begin{bmatrix} c_1(\omega_i)\mu_{tgr} + c_2(\omega_i) \\ \sigma_1(\omega_i)^2\sigma_{tgr}(\omega_i)^2 + 2\left[c_2(\omega_i) + \sigma_1(\omega_i)\sigma_2(\omega_i)\right]\mu_{tgr}(\omega_i) + \sigma_2(\omega_i)^2 \end{bmatrix} \tag{10}$$

$$\nabla\mathbf{hi} = \begin{bmatrix} \mu_{tgr}(\omega_i) & 1 & 0 & 0 \\ 0 & 2\mu_{tgr}(\omega_i) & 2\left[\sigma_1(\omega_i)\sigma_{tgr}(\omega_i)^2 + \sigma_2(\omega_i)\mu_{tgr}(\omega_i)\right] & 2\left[\sigma_1(\omega_i)\mu_{tgr}(\omega_i) + \sigma_2(\omega_i)\right] \end{bmatrix} \tag{11}$$

After applying the extended Kalman filter, the parameters $c1(\omega_i)$, $c2(\omega_i)$, $\sigma_1(\omega_i)$ and $\sigma_2(\omega_i)$ are identified. A new sample of differential mean and differential variance of group delay time can be obtained respectively. Comparison of the simulated mean and variance of group delay time and the observed mean and variance of group delay time of Hyogo-Nanbu Earthquake are shown in Figure 4.The simulated group delay time characteristics are in good agreement to those of the observed ones.

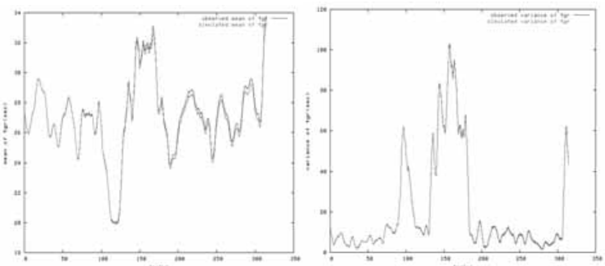

Figure 4 Comparison of a simulated mean and variance of group delay time and the observed ones

4. Conclusion

A method to simulate the group delay time characteristics of earthquake motion from the observed earthquake motion data based on the concept of stochastic different equation was developed. Extended Kalman filter effectively identify the unknown parameters of group delay time modeled by a linear stochastic differential equation model. As a conclusion, group delay time can be modeled by a linear stochastic differential equation for simulating phase characteristic of earthquake motion.

5. References

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