# Performance Evaluation of Link-Bearing Connection of Yokohama-Bay Bridge during Earthquake using State-Space Based System Identification

University of Tokyo	
University of Tokyo	

Student Member Fellow

## Introduction

The earthquake resistance of Yokohama bay bridge was carefully reviewed in design. To limit the effect of large inertia force from superstructure into substructures, the towers and end-piers are connected to the main girder by link-bearing connection. These links are designed to function as perfectly hinged connection during bridge movement in the longitudinal axis. Hence, this prevents excessive inertia force from superstructure to be transmitted to the end-piers and other sub-structures. This reports describes the investigation of performance evaluation of the link-bearing connection using system identification technique. Earthquake responses from various ground motion are used for modal analysis using time domain state-space system identification to investigate the performance of the link-bearing connection during earthquake.

## Methodology

Using a finite dimensional, discrete-time, linear, timeinvariant, state variable dynamical system, the earthquake-induced equation of motion of structure can be expressed as:

> $\mathbf{x}(\mathbf{k+1}) = [\mathbf{A}] \mathbf{x}(\mathbf{k}) + [\mathbf{B}] \mathbf{z}(\mathbf{k})$ (1)  $\mathbf{y}(\mathbf{k}) = [\mathbf{R}] \mathbf{x}(\mathbf{k}) + [\mathbf{D}] \mathbf{z}(\mathbf{k})$

Vector z(k) denotes the  $q \ge 1$  input acceleration of ground motion and y(k) denotes the  $m \ge 1$ , structural acceleration responses as outputs. The integer k=0, 1, 2, ... l denotes the time-step number i.e.  $x(k+1)=x(k(\Delta t) + \Delta t)$ , with  $\Delta t$  being the time interval. Further expanding the above state-space equations into a matrix equation of p sampled data, using the observability matrix matrix  $[\mathbf{O}_p]$  and  $[\mathbf{T}_p]$  yields,

$$Y_{p}(k) = [O_{p}]X(k) + [T_{p}]Z_{p}(k)$$
(2)

The objective of state-space system identification is to determine the unknown matrices [A], [B], [R] and [D], which are embedded in the observability matrix  $[O_p]$  and the Toeplitz matrix of system Markov parameters  $[T_p]$  from sets of input-output data. One can start by computing the matrix  $[O_p]$  and  $[T_p]$ , and later compute the state matrix [A] and output influence matrix [R] using algebraic manipulation of the observability matrix  $[O_p]$  such as:

 $[\mathbf{A}] = [O_{p}^{*}](1:(p-1)m,:)[O_{p}](m+1:pm,:) \quad (3)$ Along this line, reference [1] suggested that the observability matrix  $[\mathbf{O}_{\mathbf{p}}]$  could be obtained by factoring the information matrix  $R_{hh} = R_{yy} - R_{yz}R_{zz}^{-1}R_{yz}^{T}$ , which consists of correlation (R) of input (z) and output (y) data such follows:

Dionysius M SIRINGORINGO<sup>1)</sup>

Yozo FUJINO<sup>2)</sup>

 $R_{_{hh}} = [O_{_{p}}]\hat{R}_{_{xx}}[O_{_{p}}]^{^{T}} = H_{_{2N}}\sum_{_{2N}}^{^{2}}H_{_{2N}}^{^{T}}$ (4) This equality produces one solution each for  $[\mathbf{O}_{\mathbf{p}}] = H_{_{2N}}$ . Hence, the system matrix  $[\mathbf{A}]$  is obtained. Modal parameters of structural system can be estimated by solving the eigenvalues problem of matrix  $[\mathbf{A}]$  as  $[A][\hat{\Phi}] = [\hat{\Phi}][\tilde{\Lambda}]$ , and after transformation yields the natural frequency and modal damping ratio:

$$\omega_{i} = \sqrt{\operatorname{Re}\left(\lambda_{i}\right)^{2} + \operatorname{Im}\left(\lambda_{i}\right)^{2}} \quad \xi_{i} = \frac{-\operatorname{Re}\left(\lambda_{i}\right)}{\omega_{i}} \tag{5}$$

The mode shapes matrix in coordinate system is obtained by transforming the eigenvectors using the outputtransformation matrix  $[\mathbf{R}]$ . The complete procedure of system identification can be found in reference [2]

#### **Application to the Yokohama Bay Bridge**

For the purpose of evaluation of link-bearing connection only the first longitudinal mode from system identification is presented. Responses were obtained from 20 output sensors: 4 sensors along side the girder, 12 sensors at the both towers and 4 sensors at the both piers, all are measuring accelerations in the longitudinal (x) direction. As inputs, four triaxial sensors (x,y and z directions) at the bottom of the left-end and right-end piers and both of towers were utilized. The outputs end-piers sensors are located on the pier-cap and the girder just above the adjacent pier cap. All responses were sampled at sampling rate of 100 Hz. Responses for analysis were obtained from acceleration records of eight earthquakes recorded from 1990 to 1997. These records were divided into fourteen frames to (see Table I for detail). Modal parameters for these records were calculated separately.

## **Results of System Identification**

Three typical first longitudinal modes were obtained from system identification. The first mode was identified at around 0.128 Hz to 1.334 Hz, which is a typical first mode when compared to the finite element model [3]. This mode shows a large relative modal displacement between piercap of end-piers and the girder. This large gap indicates that a slip mechanism between pier-cap and girder has taken place during the earthquake as in the condition of a fully hinged connection. The second and third modes were identified at higher frequencies between 0.18 to 0.24 Hz. The second mode displays a mixed mechanism in which one of the

Keyword: System Identification, Earthquake-Induced System Identification, state-space realization, Yokohama Bay Bridge Address: Hongo 7-3-1, Bunkyo-ku, Tokyo 113-8656, Japan; Tel: 03-5841-6099, <u>Fax</u>: 03-5841-7454
Email: 1) dion@bridge.t.u-tokyo.ac.jp, 2) fujino@bridge.t.u-tokyo.ac.jp

No	Earthquake (Frame number)	Maximum Inp ut Acceleration (cm/s2)	Identified   Freq (Hz)	Damp Ratio (%)			Pier-Girder	relative modal
					Type of mode		displ(p)	
					left-end pier	right-end pier	lefi-end pier	right-end pier
1	February 20 <sup>th</sup> 1990 (3)	2.13	0.209	4.775	stick	slip	0.41	0.69
2	September 11 <sup>th</sup> 1996 (3)	2.21	0.249	3.480	stick	stick	-0.29	-0.24
3	December 21 <sup>st</sup> 1996 (2)	2.45	0.213	5.255	stick	stick	0.55	0.67
4	June 5 <sup>th</sup> 1990 (1)	2.71	0.202	6.334	stick	stick	-0.25	-0.31
5	February 2 <sup>nd</sup> 1992 (2)	4.16	0.223	2.751	stick	stick	0.20	-0.46
6	September 11 <sup>th</sup> 1996 (2)	5.08	0.186	5.067	stick	stick	-0.30	-0.48
7	December 21 <sup>a</sup> 1996 (1)	5.38	0.216	9.214	stick	slip	0.24	-0.82
8	July 3 <sup>rt</sup> 1995 (1)	5.62	0.221	3.614	slip	stick	0.79	-0.12
9	February 20 <sup>th</sup> 1990 (2)	7.09	0.129	11.630	stick	slip	0.20	-1.02
10	August 9th 1997 (1)	6.1	0.234	6.215	stick	stick	-0.11	0.36
11	September 11 <sup>th</sup> 1996 (1)	8.5	0.248	2.979	stick	stick	-0.29	-0.24
12	July 9th 1997 (1)	9.49	0.247	9.980	stick	stick	0.54	0.28
13	February 20 <sup>th</sup> 1990 (1)	12.04	0.128	5.203	slip	slip	0.72	0.60
14	February 2 <sup>nd</sup> 1992 (1)	13.08	0.133	4.307	slip	slip	0.80	1.18

Table 1. Identified Natural Frequencies, Damping and type of mode



(a) Typical slip-slip Mode (Earthquake 1990-02-20 Frame-1)



(b) Typical Stick-Stick Mode (Earthquake 1992-02-02 Frame-2)



(c) Typical Mixed Slip-Stick Mode (Earthquake 1995-07-03 Frame-1)

Fig.1: Three typical mode shapes in longitudinal direction

end-pier slips while the other one remains closely connected with the girder. In the third mode, smaller relative modal displacement between pier cap and girder was observed at the both of end-piers. In this mode both of end-piers remain stick or closely connected to the girder, which indicates that the hinged mechanism of link bearing has yet to take place. The third mode suggests a stiffer connection and consequently, has a higher natural frequency.

To quantify the relative modal displacement between pier-cap and the adjacent girder a normalized relative modal displacement index ( $\phi$ ) is calculated for each endpier. This index is defined as:

$$\varphi = \phi_{girder} - \phi_{pier-cap} \tag{6}$$

where the  $\phi$  denotes the modal displacement normalized to unity for maximum value. When the pier-cap and girder are largely separated the index value is closer to unity, otherwise its value is close to zero. The results of system identification from 14 frames of earthquake are listed in Table I. The last two columns list the index of relative modal displacement between pier-caps and girder. These values determine the condition of end-pier and girder connection and thus the types of identified mode either slip or stick mode. Modes with the value of  $\phi$  larger than 0.6 is considered as slip mode.

#### Discussion

Based on system identification, the following conclusions are drawn:

- 1. During small earthquake the link-bearing connection has yet to function as a full-hinged connection. Therefore a stiffer connection with higher mode was observed. The mixed slip-stick mode was observed during smaller and moderate earthquake. The full-hinged type of connection at both of end-piers was only observed during large earthquake. This slip-slip mode suggests that link-bearings have performed as its intended function.
- 2. This investigation shows that during earthquake connection might not perform as it is predicted (fully hinged) or as it is modeled in finite-element. And this deviation or unwanted mechanism would have not been discovered without the vibration based system identification.

#### <u>References</u>

1. Juang, J.N. System Realization Using Information Matrix, Journal of

Guidance, Control, and Dynamics, 20(3), May-June, 492-500 (1997)

2. Siringoringo D., Fujino Y. (2005), "System Identification of the Yokohama-Bay Bridge using Earthquake Record by State-Space Realization" *submitted to SHMI Journal*,

3. The Yokohama Bay Bridge Published by The Metropolitan Expressway Public Corporation, Tokyo Japan, 164-172 (1991)