The Definition of Isotropy and the Generalized Rotational Hardening of Soils

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1. Introduction

The consideration of anisotropy is of importance for the description of deformation of materials. However, the distinction between the isotropy and the anisotropy, i.e. the *definition of isotropy* would not have been explained in any literature but the confusion between the isotropy of mechanical response and the material-frame indifference requiring that a constitutive equation has to be described by the isotropic tensor function is seen often. Then, the definition for the isotropy of mechanical response is first given in this article. The anisotropy for the plastic deformation behavior is described through the anisotropy of yield surface. The generalized anisotropic yield surface rotated around the central axis is given for soils, which is called the *generalized rotational hardening*.

2. Definition of isotropy

Due to the *principle of material-frame indifference,* constitutive equations have to be described by the scalar- or tensor-valued isotropic tensor function **f** fulfilling

$$\mathbf{Q}[\mathbf{f}(\boldsymbol{\sigma},\boldsymbol{\varepsilon};\overset{\circ}{\boldsymbol{\sigma}},\overset{\circ}{\boldsymbol{\varepsilon}},\mathbf{S}_i)]=\mathbf{f}(\mathbf{Q}[\boldsymbol{\sigma}],\mathbf{Q}[\boldsymbol{\varepsilon}],\mathbf{Q}[\overset{\circ}{\boldsymbol{\sigma}}],\mathbf{Q}[\overset{\circ}{\boldsymbol{\varepsilon}}],\mathbf{Q}[\mathbf{S}_i]), \tag{1}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are stress and strain measures, respectively, $\overset{\circ}{\boldsymbol{\sigma}}$ is proper corotational stress rate, $\overset{\circ}{\boldsymbol{\varepsilon}}$ is proper strain rate measure, and $\mathbf{S}_i$ denotes collectively the tensor-valued internal variables. **Q** is an arbitrary orthogonal tensor, and **Q**[ ] designates

$$(\mathbf{Q}[\mathbf{T}])_{p_1p_2\cdots p_m}=Q_{p_1q_1}Q_{p_2q_2}\cdots Q_{p_mq_m}T_{q_1q_2\cdots q_m}, \tag{2}$$

for an arbitrary *n*th-order tensor **T**.

Constitutive equations fulfilling the isotropy of mechanical response have to be independent of the rotations of directions of only the stress (rate) and strain (rate) and thus it has to fulfill the relation

$$\mathbf{Q}[\mathbf{f}(\boldsymbol{\sigma},\boldsymbol{\varepsilon};\overset{\circ}{\boldsymbol{\sigma}},\overset{\circ}{\boldsymbol{\varepsilon}},\mathbf{S}_i)]=\mathbf{f}(\mathbf{Q}[\boldsymbol{\sigma}],\mathbf{Q}[\boldsymbol{\varepsilon}],\mathbf{Q}[\overset{\circ}{\boldsymbol{\sigma}}],\mathbf{Q}[\overset{\circ}{\boldsymbol{\varepsilon}}],\mathbf{S}_i) \tag{3}$$

which means that internal variables $\mathbf{S}_i$ have to be involved in scalar forms.

3. Isotropy for plastic deformation: Yield condition

Anisotropy in a plastic deformation of materials can be described by a yield condition. A yield condition is generally described by the equation

$$f(\boldsymbol{\sigma},\ \mathbf{S}_i)=F(H), \tag{4}$$

where H is the isotropic hardening variable. Due to Eq. (3) the isotropic yield condition has to reduce to the equation

$$f(\boldsymbol{\sigma})=F(H). \tag{5}$$

2. Anisotropy of soils: Generalized rotational hardening

As was interpreted in the previous article (Hashiguchi, 2000), the kinematic hardening is not applicable to pressure-dependent materials but the rotation of yield surface (Sekiguchi and Ohta, 1977), i.e. the *rotational hardening* has to be incorporated as follows:

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$$f(p, \hat{\chi}) = F(H), \tag{6}$$

where

$$p \equiv -\frac{1}{3}\mathrm{tr}\,\boldsymbol{\sigma}, \quad \boldsymbol{\sigma}^* \equiv \boldsymbol{\sigma} + p\mathbf{I}, \quad \boldsymbol{\eta} \equiv \frac{\boldsymbol{\sigma}^*}{p}, \tag{7}$$

$$\hat{\boldsymbol{\eta}} \equiv \boldsymbol{\eta} - \boldsymbol{\beta}, \quad \hat{\chi} \equiv \frac{\|\hat{\boldsymbol{\eta}}\|}{\hat{m}}. \tag{8}$$

$\boldsymbol{\beta}$ is the second-order tensor the evolution rule of which was given Hashiguchi (1994) or Hashiguchi and Chen (1998). $\hat{m}$ is the function of

$$\sin 3\hat{\theta}_\eta \equiv -\sqrt{6}\frac{\mathrm{tr}\,\hat{\boldsymbol{\eta}}^3}{\|\hat{\boldsymbol{\eta}}\|^3}, \tag{9}$$

i.e.

$$\hat{m} = \hat{m}(\sin 3\hat{\theta}_\eta). \tag{10}$$

The simplest equation of $\hat{m}$ fulfilling the convexity of yield surface is given as follows (Hashiguchi, 2002):

$$\hat{m} = \frac{14\sqrt{6}\sin\phi_c}{(3-\sin\phi_c)(8-\sin 3\hat{\theta}_\eta)}. \tag{11}$$

ϕ_c is the angle of internal friction describing the inclination of critical state line in the (p, $\|\boldsymbol{\sigma}^*\|$) plane for the triaxial compression..

The rotation of the central axis of yield surface can be described by Eq. (6) which corresponds to the translation of the axis of yield surface in metals as interpreted by Hashiguchi (2001). However, the rotation of yield surface around the central axis cannot be described by Eq. (6), while it is not necessary for metals the yield surface of which has a circular section in the π-plane.

A more generalized anisotropy of soils is given extending the function $\hat{m}$ in Eq. (10) so that the yield surface rotates also around the central axis, describing the influence of the Lode's angle, as illustrated in Fig. 1 and it can be realized simply by replacing $\hat{\theta}_\eta$ to $\hat{\theta}_\eta - \psi$ in the function $\hat{m}$, i.e.

$$\hat{m} = \hat{m}\{\sin 3(\hat{\theta}_\eta - \psi)\}, \tag{12}$$

where ψ ($(0 \le \psi \le \pi/6)$ is the material constant describing the anisotropy.

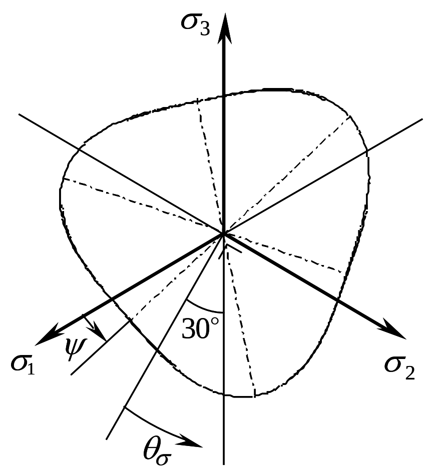

Fig. 1. Anisotropic yield surface of soils in the π-plane.

($\sin 3\theta_\sigma \equiv -\sqrt{6}\mathrm{tr}\,\boldsymbol{\sigma}^{*3}/\|\boldsymbol{\sigma}^*\|^3$)

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