Further investigation of the *J*-integral in piezoelectric ceramics

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1. Introduction:

Since piezoelectric materials show brittle nature from the mechanical point of view and susceptible to fracture when a macrocrack is formed, the understanding of the fracture behavior of piezoelectric ceramics is of great importance. Thereby it received considerable attention in the recent years. Among them, the *J*-integral, which is attractive in the fracture analysis for isotropic materials, has been given the physical meaning – the total potential energy release rate (TPERR) in piezoelectric materials by Suo [Suo et al. 1992]. In this paper, we studied the *J*-integral for piezoelectric materials further and get the conclusion that the *J*-integral is not suitable for describing the fracture behaviors under combined mechanical-electric loading conditions although it does has a clear physical meaning.

2. Analysis:

The *J*-integral was defined for the plane fracture problem in anisotropic piezoelectric materials as follows [Suo et al. 1992]:

$$J = \int_C \frac{1}{2}(\sigma_{ij}\varepsilon_{ij} - D_i E_i)dx_2 - n_i\sigma_{ip}\frac{\partial u_p}{\partial x_1}ds - n_i D_i \frac{\partial \phi}{\partial x_1}ds \tag{1}$$

which has a clear physical significance as the total potential energy release rate. Here $\sigma_{ij}, \varepsilon_{ij}, D_i, E_i, u_p, \phi$ are stress, strain, electric displacement, electric field, displacement and electric potential respectively in a coordinate system (x_1, x_2). n_i is the outward unit vector normal to contour C. The *J*-integral in anisotropic piezoelectric materials can be formulated as [Suo et al. 1992]:

$$J = \tfrac{1}{4}(\mathbf{K}^R)^{\mathrm{T}}\mathbf{H}\mathbf{K}^R \tag{2}$$

where the matrix $\mathbf{H}$ is a matrix related to the piezoelectric material constants. $\mathbf{K}^R$ is the general stress intensity factor vector of the right crack tip, i.e., $\mathbf{K}^R = [K_{II}^R \quad K_I^R \quad K_e^R]^T$.

In this paper, we adopt the Pseudo-Traction Electric Displacement Method (PTEDM) to calculate the general stress intensity factors $\mathbf{K}^R$ and the *J*-integral [Chen, et al. 1999]. The model for this method was shown in Fig.1, in which the pseudo-tractions (*P*, *Q*) and the pseudo-electric displacement (*D*) are the unknowns. After considering the traction-free and charge-free conditions on the crack faces, we can get a system of integral equations with the aid of superposition principle. After solving these equations numerically, we can calculate the general stress intensity factors $\mathbf{K}$ in terms of the pseudo-tractions and electric displacements, i.e., P, Q and D. So, the *J*-integral will be obtained by Eq. (2) without any difficulty.

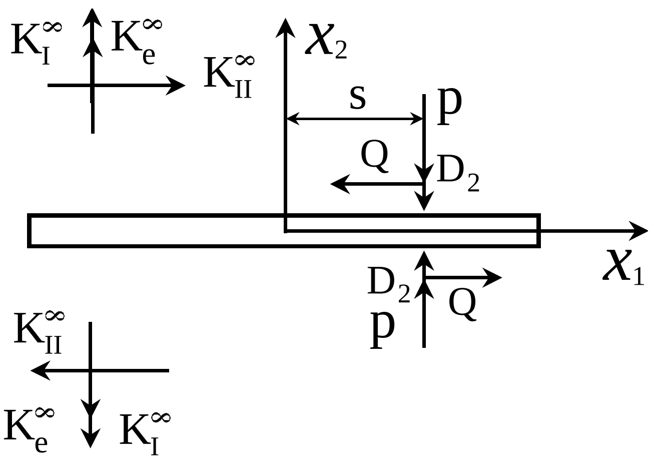

Fig.1 Calculating model for a crack in piezoelectric ceramic

Assume a crack to be normal to the x_2-axis and to be under only the mechanical loadings. In the next section, the results are normalized by the *J*-integral of this assumed crack, denoted by J_n as follows:

$$J_n = \tfrac{1}{4}(\mathbf{K}^R)^{\mathrm{T}}\mathbf{H}\mathbf{K}^R \tag{3}$$

Key words: *J*-integral, piezoelectric materials, total potential energy release rate, general stress intensity factor.
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3. Numerical example:

In this section, under plain strain conditions, one example was given to study the *J*-integral in the analysis for infinite piezoelectric materials. The **PZT-4** ceramic is chosen as the piezoelectric material under consideration. The poling direction of the **PTZ-4** ceramic is assumed to be normal to the x_1-axis. This example is concerned with a single finite crack problem. The crack is assumed to be normal to the x_2-axis and has the length *2a*. The combined mechanical-electric loading conditions are considered in this example, i.e., $\sigma_{12}^{\infty} = \sigma_{32}^{\infty} = 0$, $\sigma_{22}^{\infty} > 0$, $D_2^{\infty} = 10^{-8}\sigma_{22}^{\infty}$ CN $^{-1}$.

Then, when the normalized electric loading (i.e., $D_2^{\infty} / (10^{-8}\sigma_{22}^{\infty})$) changes from –1 to 1, the values of the normalized *J*-integrals can be obtained as Fig.2:

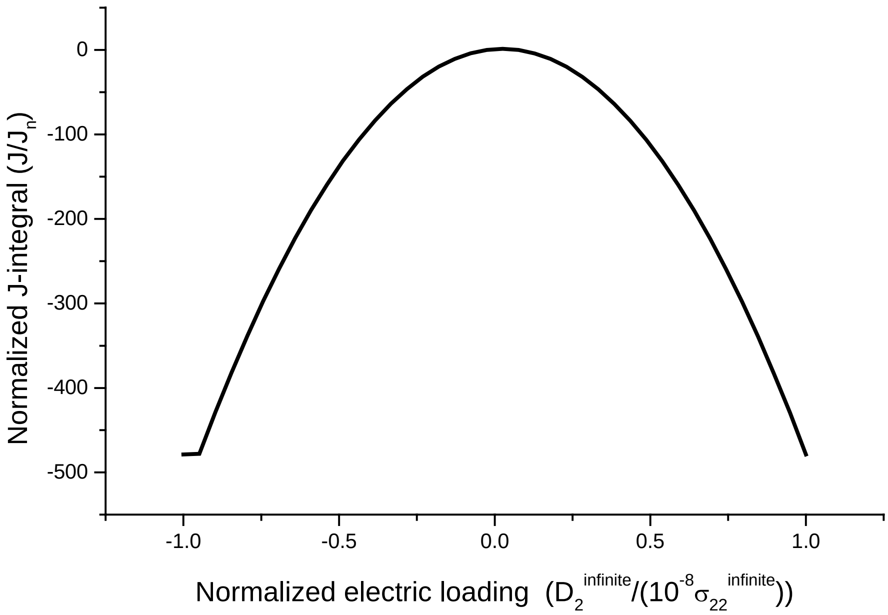

Fig. 2 Normalized *J*-integrals against the changed electric loadings

In the results, when the electric loading is zero, the normalized *J*-integral equals to 1. However, in **Fig.2**, we can see that the normalized *J*-integrals are always negative values for large level electric loadings and change significantly, which means that the crack has obvious effects on the piezoelectric material when electric loading is applied to the material. This phenomenon, however, will lead to a wrong physical conclusion, because the *J*-integral not only contains the released mechanical potential energy but also includes the released electric potential energy and the latter one cannot describe the fracture behavior in piezoelectric material. Thereby, the *J*-integral is not suitable for describing the fracture behaviors for combined mechanical-electric loading conditions, i.e., it can be used to analyze the fracture behavior for piezoelectric materials only under the pure mechanical loading conditions.

4.Conclusions:

From the analyses above, we can find that *J*-integral contains both the released mechanical potential energy and the released electric potential energy when electric loading is applied. So, it can be concluded that, for piezoelectric materials, the *J*-integral is not suitable for describing the fracture behaviors under combined mechanical-electric loading conditions although it is one of the most important parameters in fracture mechanics.

References:

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