2-D ELASTIC ANALYSIS USING THE HEXAGONAL SHAPE APPLIED ELEMENT METHOD

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1. INTRODUCTION

In order to mitigate the casualties due to structural collapse during an earthquake, collapse behavior of each structure should be investigated. This procedure inevitably requires the numerical model to predict the behavior of structure from the initial state to a total collapse. In present, there are several methods developed for collapse analysis however these models usually require a large amount of time for simulation especially when cracking propagation is involved. While the continuum approach e.g. Finite Element Methods (FEM) attempts to reduce calculation time by minimizing the time-consumable remeshing process, the discrete approach does not require this. Thus it has an advantage in time reduction compared to the continuum approach. Applied Element Method[1)] (AEM) is one of the discrete approaches used for collapse analysis. The advantage of AEM is that it can simulate structural behavior from an elastic range to total collapse with reliable accuracy within reasonable CPU time. However, numerical Poisson's ratio (ν) used in the previous version of AEM is limited to the behavior just before crack, which can lead to some errors when the model starts cracking especially when the domain space is large and crack is concentrated in only some portion of the domain space. This paper proposes a new method for representing elastic behavior in AEM. The method is based on equivalent continuum concept first proposed by Morikawa et al.[2)]. The results obtained from hexagonal shape AEM shows an excellent agreement with the continuum with the same elastic constant and the elastic behavior of hexagonal shape AEM was verified.

2. ELEMENT FORMULATION

In AEM, material is modeled as an assemblage of rigid particles interconnected along their boundaries through flexible interfaces. Two types of springs (normal and shear springs) are assumed to be distributed continuously over the boundary of the two elements as shown in Fig. 1. The system equilibrium equations are formed from each two-particle subassemblage in the domain, as outlined below.

Considering a two-particle subassemblage shown in Fig. 2, each rigid particle has two translational and one rotational degrees of freedom defined at particle centroid. Assuming small rotations, motion at any points (x,y) of a rigid body can be defined for element 1 and 2 as

$$u_1=u_{c1}-\theta_1(y-y_{c1}) \qquad v_1=v_{c1}+\theta_1(x-x_{c1})$$
$$u_2=u_{c2}-\theta_2(y-y_{c2}) \qquad v_2=v_{c2}+\theta_2(x-x_{c2}) \qquad (1)$$

where u, v, θ are two translational displacements and rotation in global coordinate x and y, respectively. Subscripts 1 and 2 define the element number and subscript c specifies the value at the particle centroid. Point p on the boundary surface is separated and defined by p' and p'' after deforming (Fig. 3). The relative displacement vector which also defines spring deformation in global coordinate of the point p can be defined as

$$\{\boldsymbol{\delta_g}\}=\overrightarrow{p'p''}=\begin{Bmatrix}\delta_x\\ \delta_y\end{Bmatrix}=\begin{Bmatrix}u_2-u_1\\ v_2-v_1\end{Bmatrix} \qquad (2)$$

Substituting Equation (1) into (2) and rotating the displacement to the local coordinate paralleled to element surface, we can obtain the relationship between spring deformation in local coordinate and particle displacement in global coordinate as

$$\{\boldsymbol{\delta_l}\}=[\mathbf{R}][\mathbf{B}]\{\boldsymbol{u}\} \qquad (3)$$

where $\{\boldsymbol{\delta_l}\}^{\mathrm{T}}=[\delta_n,\delta_t]$ in which δ_n and δ_t are normal and shear deformation of spring, respectively. Rotational matrix $[\mathbf{R}]=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, deformation-displacement relationship in global coordinate

$$[\mathbf{B}]=\begin{bmatrix}-1 & 0 & (y-y_{c1}) & 1 & 0 & -(y-y_{c2})\\ 0 & -1 & -(x-x_{c1}) & 0 & 1 & (x-x_{c2})\end{bmatrix} \text{ and}$$

$\{\boldsymbol{u}\}^{\mathrm{T}}=[u_{c1},v_{c1},\theta_1,u_{c2},v_{c2},\theta_2]$. Based on the above preliminaries, the strain energy due to spring deformation on the boundary line S can be given as

$$\boldsymbol{\nu}=\frac{1}{2}\int\{\delta_l\}^{\mathrm{T}}[\mathbf{D}]\{\delta_l\}\,\mathbf{d\,S} \qquad (4)$$

where the constitutive relationship $[D]=\mathrm{Diag}[k_{ni},k_{si}]$ in which k_{ni} and k_{si} is stiffness of normal and shear spring number i's, respectively. Applying (3) into (4), we have:

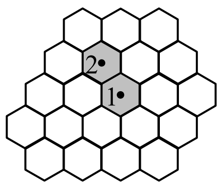

Fig.1 Hexagonal AEM

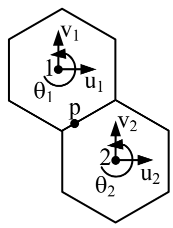

Fig.2 Two-particle assemblage and their degree of freedom

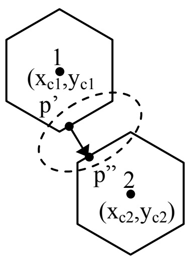

Fig. 3 Two-particle assemblage after being deformed

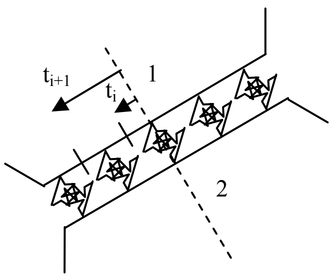

Fig.4 Normal and shear springs at element boundary

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$$\boldsymbol{\nu} = \frac{1}{2}\{\delta_i\}^{\mathrm{T}}[\mathbf{K}]\{\delta_i\} \tag{5}$$

where $[\mathbf{K}] = \sum_{i=1}^{n}\int_{t_i}^{t_{i+1}}[\mathbf{R}]^{\mathrm{T}}[\mathbf{B}]^{\mathrm{T}}[\mathbf{D}][\mathbf{B}][\mathbf{R}]dS$ is the stiffness matrix due to all springs on the boundary. t_i and t_{i+1} indicate the initial and last points of the boundary portion *i*, respectively (Fig. 4). By applying Castigliano's theorem to (5), stiffness equation can be derived as

$$\{\boldsymbol{r}\} = \frac{\partial \mathbf{V}}{\partial \mathbf{u}} = [\mathbf{K}]\{\mathbf{u}\} \tag{6}$$

where **{r}** contains the generalized force components associated with each displacement vector $\{\boldsymbol{u}\}$.

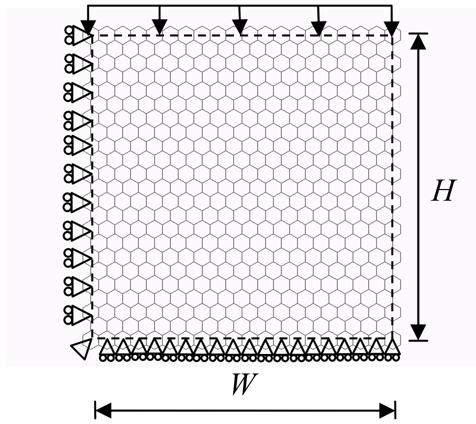

Fig.5 Axial compression of a block of elastic material

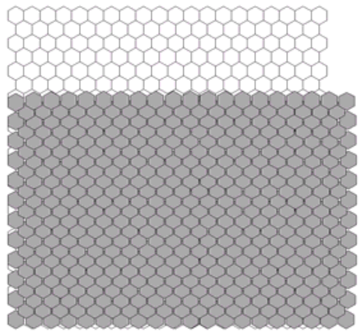

Fig.6 A deformed mesh under uniform axial loading

3. EQUIVALENT CONTINUUM

Relationship between discrete constants k_{ni} and k_{si} and the elastic properties in this study follows the equivalent continuum method proposed by Morikawa et.al.[2]. The method employs the equivalence of strain energy between discrete system and continuum and the advantage of close-pack circular discrete element geometry. To apply with AEM, it was found in the formulation process that this relationships is almost the same as proposed in Morikawa et.al.[2] but times with square root 3. Therefore, this relationship for AEM is defined as

Plane stress: $$k_{ni} = \frac{E \cdot t}{(1-\nu)\cdot d}, k_{si} = \frac{E \cdot t \cdot (1-3\nu)}{(1-\nu^2)\cdot d} \tag{7}$$

Plane strain: $$k_{ni} = \frac{E \cdot t}{(1-2\nu)(1+\nu)\cdot d}, k_{si} = \frac{E \cdot t \cdot (1-4\nu)}{(1-2\nu)(1+\nu)\cdot d} \tag{8}$$

where *E* is elastic modulus, ν= Poisson's ratio, *t* = material thickness perpendicular to plane and *d* = distance between two particles. It should be noted that the Poisson's ratio is limited from -1 to 0.33 for plain stress and -1 to 0.25 for plain strain to prevent a negative value of tangential stiffness.

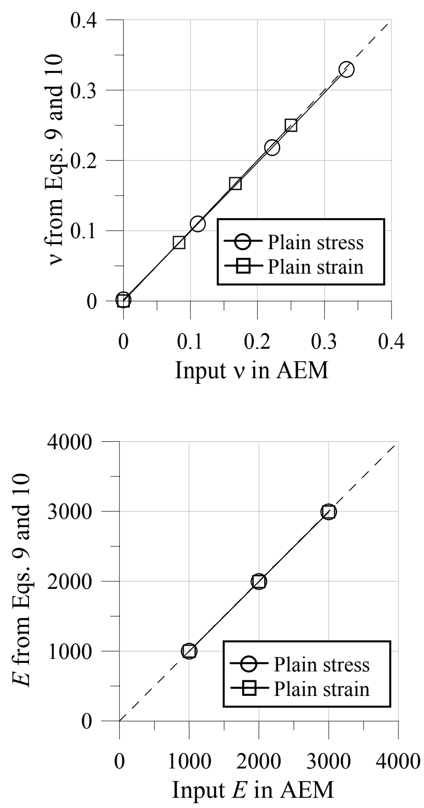

Fig.7 Plot between ν and *E* from AEM and elastic theory

4. VERIFICATION

A mesh of width *W*= 1.9, height *H*=1.9 and *t*=1 units was subjected to axial compressive stress of =278 unit. The boundary condition was shown in Fig. 5. ν varied from -1 to 0.33 in plain stress and -1 to 0.25 in plain strain, respectively. *E* varied from 1000 to 3000. Based on obtained result, ν and *E* that would have resulted in those deformations according to elastic theory in plain stress and plain strain were calculated using Eqs. 9 and 10. The elastic constants from the model were compared to the analytical solutions (Fig 7). The plots show an excellent agreement between numerical model and analytical solution. Moreover, the mesh deformation was compared to analytical result and an excellent agreement was also found.

Plane stress: $$\nu = -\frac{\delta_y}{\delta_x}\frac{W}{H}, E = \frac{\sigma}{\delta_y}H \tag{9}$$

Plane strain: $$\nu = \frac{-H/W}{\delta_y/\delta_x - H/W}, E = \frac{\sigma}{\delta_y}(1-\nu^2)H \tag{10}$$

5. CONCLUSION

The elastic behavior of hexagonal shape AEM is verified. It was found that AEM can resemble behavior of the continuum system with the same elastic constants. It should be noted that the ν in this model is limited from -1 to 0.33 for plain stress and -1 to 0.25 for the plain strain analysis. This is enough to represent the Poisson's ratio of brittle material such as concrete, rock and some kind of soil and metal. The next step of the study is to verify the non-linear and dynamic behavior.

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