Application of Shell Equations Based Shape Function for Free Vibration Analysis of Cylindrical Shells on Elastic Foundation

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1. Introduction

Cylindrical shells are widely used in many engineering fields because of their strength and effectiveness. Some of them are laid on the foundation. Yang [1] has investigated the whole buried shells under seismic loading. Free vibration analysis of whole buried shells has been studied by Paliwal *et.al* [2], while Amabili [3] has investigated the free vibration of shells which are surrounded partially by elastic bed based on the Rayleigh-Ritz method. Lakis and Sinno [4] have developed the combined formulation of finite element and classical shell theory which is applied for free vibration of empty and liquid filled shells. This paper presents the application of such formulation on the free vibration of shells on elastic foundation. Comparison between classical finite element using polynomial [5] and the present one is given.

2. Model and Formulation

The shell considered is an isotropic thin elastic shell with Young's modulus E, Poisson's ratio υ, radius of the middle surface R, thickness h and length L. Foundation is represented by continuous elastic (axial, circumferential, radial and rotational) springs on a limited arc which corresponds to an angle $\varphi_1 + \varphi_2$. Geometry of the structure and generalized model with reference direction are shown in Fig. 1, where K_u, K_v, K_w and K_β denote the associated directional spring coefficients.

Governing equations of empty shell without the existence of foundation are given by Eq. 1., where L is the corresponding differential operator. The displacements of a point on the middle surface in axial, circumferential and radial direction are indicated by u, v and w, respectively. While rotation angle β is defined as the first derivative of w with respect to x. The displacement functions are given in Eq. 2.

$$L_1(u,v,w) = 0 \; ; L_2(u,v,w) = 0 \; ; L_3(u,v,w) = 0 \tag{1}$$

$$\begin{aligned} u(x,\theta) &= \sum_{m=0}^{M} \left\{ U_m^{\mathrm{S}}(x)\cos(m\theta) + U_m^{\mathrm{U}}(x)\sin(m\theta) \right\} \\ v(x,\theta) &= \sum_{m=0}^{M} \left\{ V_m^{\mathrm{S}}(x)\sin(m\theta) + V_m^{\mathrm{U}}(x)\cos(m\theta) \right\} \\ w(x,\theta) &= \sum_{m=0}^{M} \left\{ W_m^{\mathrm{S}}(x)\cos(m\theta) + W_m^{\mathrm{U}}(x)\sin(m\theta) \right\} \end{aligned} \tag{2}$$

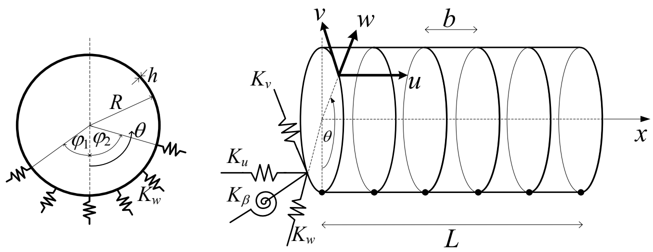

Fig. 1. Geometry and generalized model

Superscript S and U refer to symmetrical and unsymmetrical systems. Appearing shape functions in Eq. 1. for symmetrical system are represented by

$$U_m^{\mathrm{S}}(x) = A_m^{\mathrm{S}}\, e^{\mu x/R} \; ; V_m^{\mathrm{S}}(x) = B_m^{\mathrm{S}}\, e^{\mu x/R} \; ; W_m^{\mathrm{S}}(x) = C_m^{\mathrm{S}}\, e^{\mu x/R} \tag{3}$$

where A, B and C are constants. Similar expression for unsymmetrical system can be considered. Substituting Eq. 2 and Eq. 3 into Eq. 1 and then for a non trivial solution determinant of the resulting matrix should be zero. This produces an eighth order characteristic polynomial which can be solved to find eight characteristic values, μ. Eq. 1. can be rewritten in the matrix form (including β) as Eq. 4. Strain $\boldsymbol{\varepsilon}$ and stress $\boldsymbol{\sigma}$ vectors are given by Eq. 5, where $\boldsymbol{\varepsilon}$ is composed of three strains and three curvature changes of the middle surface. $\boldsymbol{\sigma}$ is composed of three in-plane and three out-of-plane stress resultants. $\mathbf{P}$ is an elasticity matrix.

$$\boldsymbol{\delta} = \{u \quad v \quad w \quad \beta\}^{\mathrm{T}} = \sum_{m=0}^{M}\left(\mathbf{N}_m^{\mathrm{S}}\, \boldsymbol{\delta}\mathbf{e}_m^{\mathrm{S}} + \mathbf{N}_m^{\mathrm{U}}\, \boldsymbol{\delta}\mathbf{e}_m^{\mathrm{U}}\right) = \mathbf{N}\, \boldsymbol{\delta}\mathbf{e} \tag{4}$$

$$\boldsymbol{\varepsilon} = \{\varepsilon_x \;\; \varepsilon_\theta \;\; 2\varepsilon_{x\theta} \;\; \chi_x \;\; \chi_\theta \;\; 2\chi_{x\theta}\}^{\mathrm{T}} = \mathbf{B}\, \boldsymbol{\delta}\mathbf{e} \; ; \boldsymbol{\sigma} = \{N_x \;\; N_\theta \;\; N_{x\theta} \;\; M_x \;\; M_\theta \;\; M_{x\theta}\}^{\mathrm{T}} = \mathbf{P}\, \boldsymbol{\varepsilon}, \tag{5}$$

Keywords: Free Vibration, Finite Element Method, Cylindrical Shells, Foundation

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The stiffness $\mathbf{K_S}$ and mass $\mathbf{M_S}$ matrix of the shell are given by Eq. 6. The stiffness matrix of the foundation $\mathbf{K_F}$ can be written as in Eq. 7, in which for this purpose, the distribution of foundation may be represented by Fourier series in circumferential direction.

$$\mathbf{K_S} = \iint_A \mathbf{B}^{\mathrm{T}} \mathbf{P}\, \mathbf{B}\, \mathrm{d}A \; ; \mathbf{M_S} = \rho h \iint_A \mathbf{N}^{\mathrm{T}} \mathbf{N}\, \mathrm{d}A, \tag{6}$$

$$\mathbf{K_F} = \iint_A \mathbf{N}^{\mathrm{T}} \mathbf{\Psi}\, \mathbf{N}\, \mathrm{d}A \text{ , where } \mathbf{\Psi} = diag(\kappa_u(\theta), \kappa_v(\theta), \kappa_w(\theta), \kappa_\beta(\theta)) \tag{7}$$

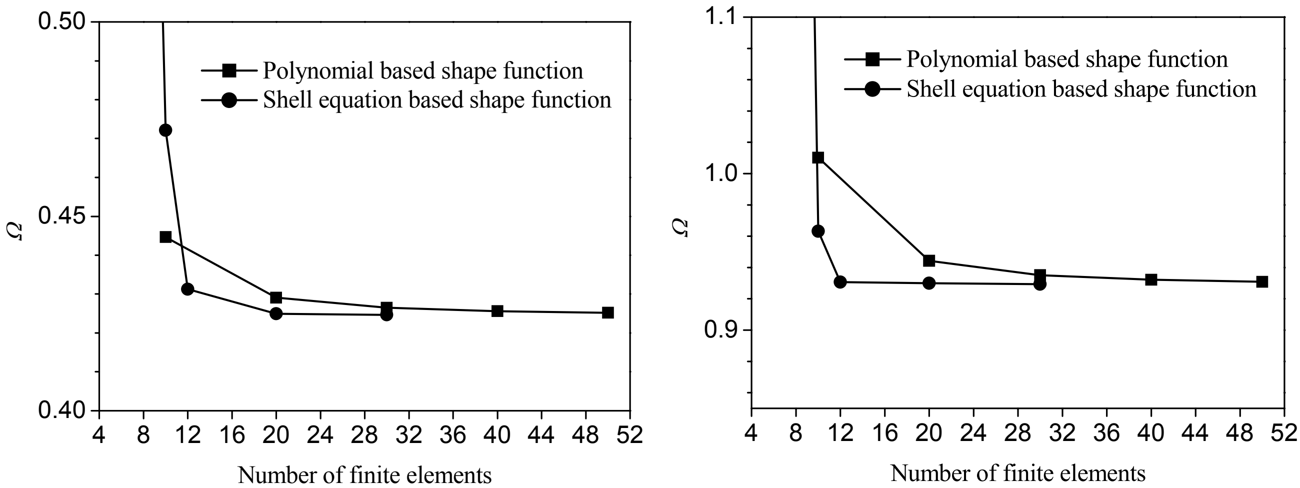
The global equation of the problem [5] is given by Eq. 8 which can be solved for the natural frequencies and modes.

$$\sum_{ns=1}^{NS}\sum_{n=0}^{N}\left[\begin{bmatrix}\mathbf{K}_{\mathbf{S}mn}^{\mathrm{SS}} & \mathbf{0}\\ \mathbf{0} & \mathbf{K}_{\mathbf{S}mn}^{\mathrm{UU}}\end{bmatrix}+\begin{bmatrix}\mathbf{K}_{\mathbf{F}mn}^{\mathrm{SS}} & \mathbf{K}_{\mathbf{F}mn}^{\mathrm{SU}}\\ \mathbf{K}_{\mathbf{F}mn}^{\mathrm{US}} & \mathbf{K}_{\mathbf{F}mn}^{\mathrm{UU}}\end{bmatrix}\right]^{ns}\begin{Bmatrix}\boldsymbol{\delta}\mathbf{e}_n^{\mathrm{S}}\\ \boldsymbol{\delta}\mathbf{e}_n^{\mathrm{U}}\end{Bmatrix}^{ns} = \omega^2\sum_{ns=1}^{NS}\sum_{n=0}^{N}\begin{bmatrix}\mathbf{M}_{\mathbf{S}mn}^{\mathrm{SS}} & \mathbf{0}\\ \mathbf{0} & \mathbf{M}_{\mathbf{S}mn}^{\mathrm{UU}}\end{bmatrix}^{ns}\begin{Bmatrix}\boldsymbol{\delta}\mathbf{e}_n^{\mathrm{S}}\\ \boldsymbol{\delta}\mathbf{e}_n^{\mathrm{U}}\end{Bmatrix}^{ns}, \tag{8}$$

$$\text{for } m = 0, 1, 2, \ldots N = M,$$

3. Numerical Results

Numerical example is presented for simply supported shell at both ends. In the analysis, only radial spring is considered ($K_u = K_v = K_\beta = 0$). These following parameters are used $\upsilon = 0.30$, $R/L = 0.05$, $R/h = 20$, $K_w L/E = 0.001$, $\varphi_1 = \varphi_2 = \pi/3$ and $M = 6$. Convergence of the first and third natural frequencies of the symmetric vibration is given in Fig. 2, where non-dimension frequency parameter $\Omega = \omega L\sqrt{\rho(1-\upsilon^2)/E}$ is plotted against the number of finite elements.



(a) First frequency (b) Third frequency

Fig. 2. Convergence of Ω

4. Conclusions

Application of shell equation based shape function for finite element analysis of cylindrical shells on elastic foundation has been shown. The shape function in axial direction is directly derived from the shell theory. Numerical studies show that the application of this formulation gives significant improvement on the convergence of the results.

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