System Identification of the Yokohama-Bay Bridge using Earthquake-Induced Record by State-Space Realization

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Introduction

A methodology to identify dynamic characteristics of linear structure based on earthquake-induced vibration data is presented in this study. The algorithm is based on the system realization using information matrix (SRIM). In this system, a state-space-based realization is utilized to identify the system observability matrix from the information matrix consisting of input-output data correlation, from which the system matrices are determined to estimate modal parameters. The methodology is applied to identify modal parameters of Yokohama-Bay Bridge.

Methodology

Consider the equation of motion for an N degree-of-freedom (DOF) linear, time invariant, viscously damped system subjected to earthquake excitation $\alpha(t)$, in the spatial coordinate and continuous time $u(t)$ as

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=-[M]\alpha(t) \qquad (1)$$

where [M],[C], and [K] are the mass, damping and stiffness matrices respectively. In a finite dimensional, discrete-time, linear, time-invariant, state variable dynamical system, this equation can be expressed as:

$$x(k+1) = \mathbf{[A]}\, x(k) + \mathbf{[B]}\, z(k) \qquad (2.a)$$

$$y(k) = \mathbf{[R]}\, x(k) + \mathbf{[D]}\, z(k) \qquad (2.b)$$

Wherein the latter expression, the vector is a $2n$ x 1 state vector consisting of displacement and velocity. Vector $\boldsymbol{z}$(k) denotes the q x 1 input acceleration of base or ground motion and $\boldsymbol{y}$(k) denotes the m x 1, structural acceleration responses as outputs. The integer $k=0,1,2\ldots l$ denotes the time-step number i.e. $\boldsymbol{x}(k+1)=\boldsymbol{x}(k(\Delta t)+\Delta t)$, with Δt being the time interval. Further expanding the above state-space equations into matrix equation of p sampled data, using the observability matrix matrix $\mathbf{[O_p]}$ and $\mathbf{[T_p]}$ yields,

$$Y_p(k)=[O_p]X(k)+[T_p]Z_p(k) \qquad (3)$$

The aim of state-space system identification is to determine the unknown matrices **[A],[B],[R]** and **[D],** which are embedded in the observability matrix $\mathbf{[O_p]}$ and the Toeplitz matrix $\mathbf{[T_p]}$ from given sets of input-output data. One can start by computing the matrix $\mathbf{[O_p]}$ and $\mathbf{[T_p]}$, and later compute the state matrix [**A**] and output influence matrix **[R]** using algebraic manipulation of the observability matrix $\mathbf{[O_p]}$ such as:

$$\mathbf{[A]}=[O_p^*](1:(p-1)m,:)[O_p](m+1:pm,:) \qquad (4)$$

Using this approach, reference [1] suggested that the observability matrix $\mathbf{[O_p]}$ could be obtained by factoring the information matrix $R_{hh}=R_{yy}-R_{yz}R_{zz}^{-1}R_{yz}^{T}$, which consists of correlation (R) of input (z) and output (y) data using singular value decomposition such follows:

$$R_{hh}=[O_p]\hat{R}_{xx}[O_p]^T=H_{2N}\,\Sigma_{2N}^2\,H_{2N}^T \qquad (5)$$

This equality produces one solution for $\mathbf{[O_p]}=H_{2N}$.

Hence, the system matrix **[A]** can be extracted using equation (4). Modal parameters of structural system can be estimated by solving the eigenvalues problem of matrix **[A],** such as:

$$[A][\hat{\Phi}]=[\hat{\Phi}][\hat{\Lambda}] \qquad (6)$$

whose eigenvalues and eigenvectors after transformation, yields the natural frequency and modal damping ratio:

$$\omega_i=\sqrt{\mathrm{Re}(\lambda_i)^2+\mathrm{Im}(\lambda_i)^2}\quad \xi_i=\frac{-\mathrm{Re}(\lambda_i)}{\omega_i} \qquad (7)$$

Keyword: System Identification, Earthquake-Induced System Identification, state-space realization, Yokohama Bay Bridge
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Mode shapes matrix in coordinate system is obtained by transforming the eigenvectors in *z*-domain into coordinate-domain using the output-transformation matrix **[R]**. The complete procedure of system identification can be found in reference. [2]

Application to the Yokohama Bay Bridge

The proposed methodology is applied to the Yokohama-Bay Bridge, a three-continuous-span cable stayed bridge in Yokohama bay area near Tokyo Japan. For this application of system identification 6 channels of inputs on two locations of tower foundation (i.e. both are in x-y-z directions) were chosen. It should be noted here that these channels were chosen instead of the free-field channels as to minimize the effect of soil-structure interaction in the analysis. Outputs consist of 19 channels in lateral-vertical and 17 channels in longitudinal-vertical directions. The ground motions used in the present study were measured from the January 20, 1990 Oshima Island. This earthquake of a magnitude of 6.5 was in 4^{th} scale of Japanese Magnitude Index (JMI) measured in Yokohama. Records were sampled at 100 Hz with the 195.5 seconds of total data length.

Results and Discussion

System identification successfully identifies fourteen modes: five bending modes in the longitudinal-vertical direction, three torsional modes and six in the lateral-longitudinal direction. List of the identified mode shapes is shown in Table I, and some of the mode shapes are shown in Figure 1. The results were also compared with the computed results from FEM and ambient vibration test [3] performed right after the completion of the structure. Table 1 shows that all the identified natural frequencies were within the range of natural frequencies from FEM model, the ambient vibration test. The proposed system identification also successfully identified several modes that were not identified by other tests. Example application to the Yokohama-Bay Bridge has demonstrated successfully the feasibility of system identification for large and complex structures system. This could lead to a better approach in modal analysis of large structure from earthquake-induced vibration.

Mode	Identified		FEM [Hz]	Ambient Test [Hz]
	Freq (Hz)	Damp. (%)		
Longitudinal-Vertical				
1st Sym Bending	0.36	5.91	0.34	0.36
1st Asym Bending	0.56	4.63	0.49	0.60
2nd Sym Bending	0.81	1.38	0.77	0.85
2nd Asym Bending	1.01	1.11	1.22	1.01
3rd Sym Bending	1.21	1.33	N/A	N/A
1st Torsional	0.87	2.24	0.95	0.88
2nd Torsional	1.38	2.68	N/A	N/A
3rd Torsional	2.50	2.02	N/A	N/A
Lateral-Vertical				
1st Sym Bending Tower-Girder same phase	0.30	5.67	0.28	0.27
1st Symmetric Bending Tower-Girder anti phase	0.44	4.66	N/A	N/A
1st Asymmetric Bending Tower anti phase	0.39	1.86	0.42	0.38
1st Asymmetric Bending Tower-Girder anti phase	0.71	4.61	0.70	0.68
1st Asymmetric Bending Tower same phase	0.94	2.34	N/A	N/A
2nd Symmetric Bending	1.07	2.14	1.08	-

Table 1. Identified Natural Frequencies and Damping

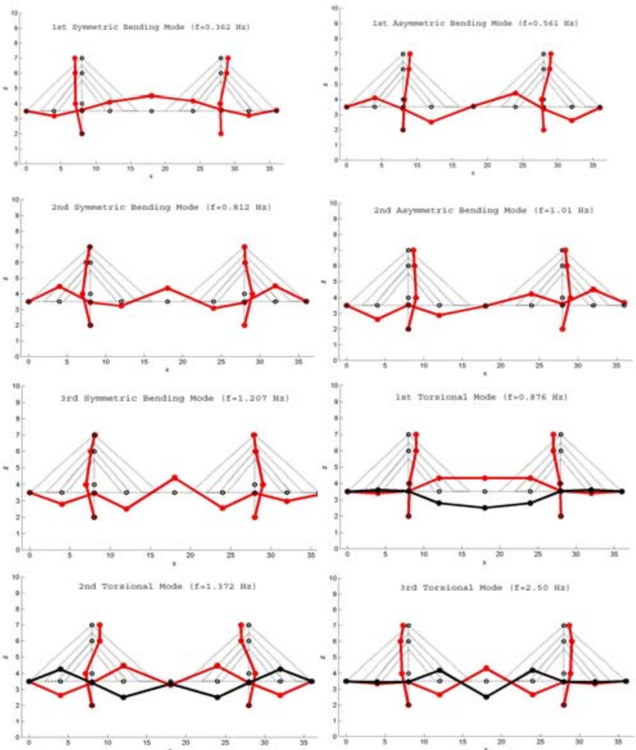

Fig.1*: Mode shapes in Longitudinal and Vertical direction*

References

1. Juang, J.N. State-Space System Realization with Input-Output Data Correlation, *NASA Technical Paper 3622.* (1997),

2. Siringoringo D., Fujino Y., "System Identification of the Yokohama-Bay Bridge using Earthquake Record by State-Space Realization" *submitted to EESD Journal* (2004),

3. The Yokohama Bay Bridge *Published by The Metropolitan Expressway Public Corporation, Tokyo Japan,*164-172 (1991)