L-integral analysis in finite brittle solids with microcracks

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1.Introduction:

The study on solids with microcracks has received considerable attentions since the existence, growth and nucleation of microcracks in solids are of significant importance in both mechanical engineering and civil engineering. Within the framework of plane, linear fracture mechanics, the conservation laws (J-integral, L-integral and M-integral) were proposed and extremely attractive, among which the L-integral was given the physical meaning as the crack rotational energy release rate by Herrmann and Herrmann [1981]. The purpose of the present work is to analyze what role the L-integral plays in finite brittle solids with microcracks. The basic idea starts from the L-integral analysis customarily used in single crack problems of infinite solids, which was regarded as a more natural description of energy release rate for plane cracks, rather than the J-integral. The analyses in finite microcracking solids should be carried out with caution. As could be imagined, the main difficulties to be overcome are concerned with the treatments of the outside boundaries. As an initial attempt, only the homogenous, plane, brittle solid is considered in this paper.

2. Analisys:

It is well known that the L-integral was defined as follows [Knowles and Sternberg (1972)]:

$$L = \oint_C e_{3ij}(w \cdot x_j \cdot n_i - T_i u_j - T_k \cdot u_{k,i} x_j)\, ds \qquad (k = 1,2) \tag{1}$$

where C is a closed contour in x_1x_2 plane; the crack faces are along the x_1-axis; w is the strain energy density; T_k is the traction acting on the outer-side of the closed contour C; u_j is displacement, $u_{k,i} = \partial u_k / \partial x_i$ (i=1,2); The L-integral has been given as the crack rotational energy release rate [Herrmann and Herrmann, 1981].

As shown in Fig.1, assume that the closed contour C encloses all microcracks completely in a finite plane solid with loading σ_0 applied on the outside boundaries.

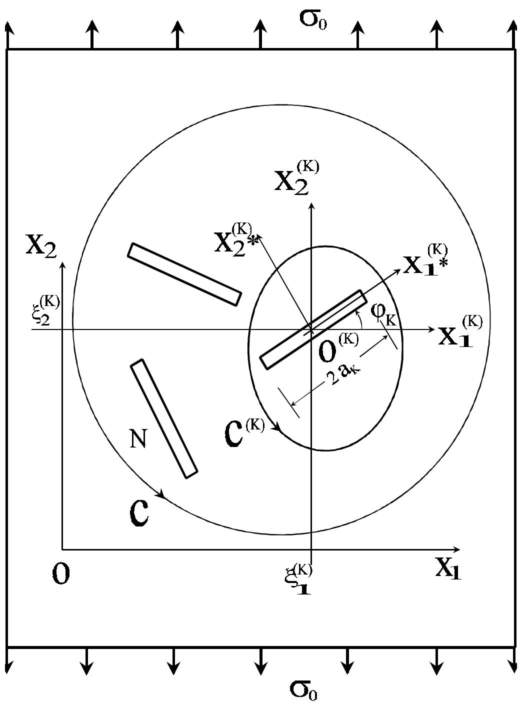

Fig.1 Interacting microcracks in a finite plane solid and the closed contours specially introduced

To the problem shown in Fig.1, the L-integral in the global system (x_1, x_2) could be evaluated as follows:

$$\begin{aligned} L &= \sum_{k=1}^{N} \{ \oint_C e_{3ij}(w \cdot x_j \cdot n_i - T_i u_j - T_k \cdot u_{k,i} x_j) ds \} \\ &= \sum_{k=1}^{N} L^{(k)}(x_1^{(k)}, x_2^{k}) + \sum_{k=1}^{N} \{ \xi_2^{(k)} J_1^{(k)} - \xi_1^{(k)} J_2^{(k)} \} \end{aligned} \tag{2}$$

where $L^{(k)}(x_1^{(k)}, x_2^{(k)})$ is considered in the local system $(x_1^{(k)}, x_2^{(k)})$ and its value could be evaluated by using Herrmann's formulation [Herrmann and Herrmann, 1981]:

$$\begin{aligned} & L^{(k)}(x_1^{(k)}, x_2^{(k)}) \\ &= -\frac{3(\kappa - 1)}{4\mu} [K_{IR}^{(k)} \cdot K_{IIR}^{(k)} + K_{IL}^{(k)} \cdot K_{IIL}^{(k)}] a_k \end{aligned} \tag{3}$$

where μ is the shear modulus of the brittle solid, $\kappa = 3 - 4\nu$ is for plane strain, ν is the poisson's ratio, a_k is the half length of the k-th crack, and subscripts R and L denote the quantities at the right tip and the left tip respectively, while subscripts I and II denote the mode I and mode II fracture, respectively. In equation (2), $J_1^{(k)}$, $J_2^{(k)}$ are the first and second components of the J_k-vector in the local system $(x_1^{(k)}, x_2^{(k)})$; $\xi_1^{(k)}$ and $\xi_2^{(k)}$ are coordinates of the origin $O(k)$ in the global system (x_1, x_2) (see Fig.1).

In this paper, the calculated values of the L-integral are normalized by:

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$$L_0 = -\frac{3(\kappa - 1)}{4\mu} \cdot \pi \cdot (\sigma_0 \cdot a_0)^2 \tag{4}$$

where σ_0 is the tension stress applied at the finite plate (see Fig.1) while a_0 is the average half length of the cracks.

Based on the analyses in this section, the *L*-integral for finite microcracking solids could be calculated by a method combined by the Boundary Element Method [Crouch and Starfield,1983] and the Pseudo-traction method[Chen Y.Z,1984].

3. Numerical results:

As shown in Fig.2, a homogeneous finite plate with two collinear interacting cracks is adopted in this example, 2*b=2,2c*=1.1 and 2a=0.5. The left crack in this problem does not rotate while the right one rotates with the angle α . The Al_2O_3 ceramic is adopted. Its shear modulus is $1.792 \times 10^{11}\ N/m^2$ while the Poisson's ratio is 0.207 . The remote tension load for this example is σ_0. Then, for different crack inclination angles α , the normalized values of the *L*-integral are obtained and plotted in Fig.3.

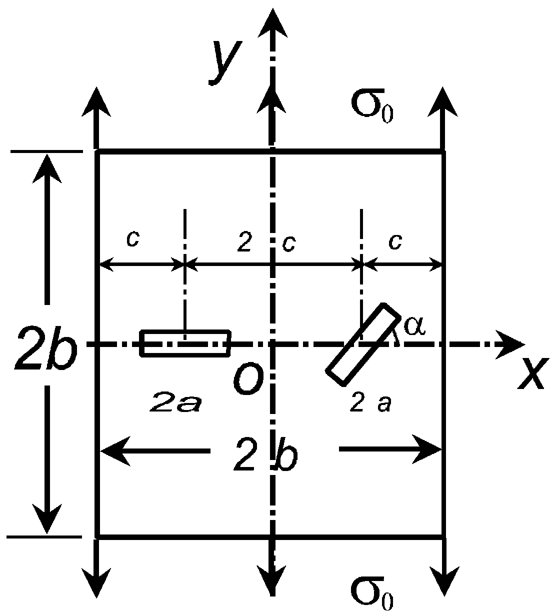

Fig.2 A finite plate with two co-linear cracks (only the right crack rotates with α).

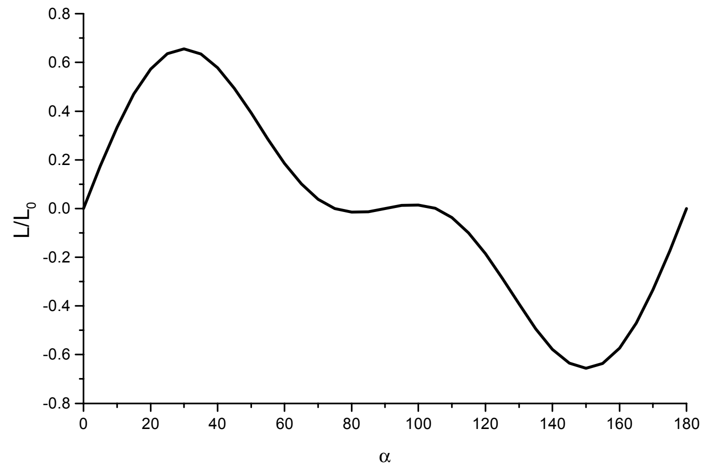

Fig.3 The normalized values of the *L*-integral versus the microcrack orientation angles α

As shown in Fig.3, the magnitudes of the *L*-integrals are symmetrical about $\alpha = 90^\circ$ while the signs of them are quite opposite. The *L*-integral values tend to the maximum when $\alpha = 30^\circ$, and the minimum when $\alpha = 150^\circ$. From the physical viewpoint, it reveals that the trend of the cracks to rotate tends to the maximum when $\alpha = 30^\circ$ and the minimum when $\alpha = 150^\circ$ due to the tension load σ_0, which means that the system is of the smallest stability when $\alpha = 30^\circ$and of the largest stability when $\alpha = 150^\circ$. In other words, the value of the *L*-integral reflects the stability of the system, which is mainly influenced by the strong interactions among the cracks and the outside boundaries.

4.Conclusions:

The *L*-integral for finite microcracking solids can be calculated numerically with the aid of both the BEM and the Pseudo-traction method. The result was discussed further with its physical meaning, from which the stabilities of the system and the interacting effects (including the shielding and the amplification effects) among the microcracks and the outside boundaries can be analyzed. Moreover, from the analysis in this paper, we can find that the different values of the *L*-integral in finite solids just represent the rotate energy release of the microcracks due to the strong interactions among the cracks and the out boundaries. It can be imagined that there exists a possibility or evidence that the *L*-integral can apply an effective evaluation of the stability of the damaged finite system, so the potential applications of the *L*-integral analysis in finite brittle solids with microcracks must be interesting and need to be investigated further.

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