Observational Method using Kalman Filtering for Land Subsidence Prediction in Bangkok

Kyoto University, Member, Hiroyasu Ohtsu
Kyoto University, Student, Sirisin Janrungautai
Kyoto University, Member, Yuzo Ohnishi
Kyoto University, Member, Kenji Takahashi

1. Introduction

Since 1950s groundwater has been used as an important source of water supply in Bangkok area, the capital of Thailand, which is located in the Lower Chao Phraya basin. At present, due to increase in groundwater extraction over the entire period, Bangkok has been facing the severe problem of land subsidence, which has intensified a lot of problems such as flooding and foundation damages (Natalaya and Phienwej, 2001). By considering land subsidence as an urgent issue, the project of management of groundwater and land subsidence was initiated by Department of Mineral Resources (DMR) and now being studied supported by Japan International Cooperation Agency (JICA) (Ramnarong, 1999). The prediction of land subsidence is one of the important tasks within the project. In this study, the autoregressive model (AR) is used to represent subsidence process and Kalman filtering is applied to identify model parameters based on observed data of land subsidence.

2. Land Subsidence in Bangkok area

The overall groundwater extraction in Bangkok area is in critical rate, over 2.0 million m^3/day in 1999. The extensive groundwater pumping since 1970s led to significant drawdown in piezometric levels of the main aquifers to 50-60 m. Consequently, land subsidence has continuously occurred, totally around 0.1-0.9 m up until now. The present rate of land subsidence is 5-20 mm/year in the central area and 25-45 mm/year around the industrial zone in the suburbs. For mitigation of land subsidence crisis, groundwater pumping rate has to be controlled to restore piezometric levels in the aquifer system. On the other hand, mitigation of prospective damages from land subsidence must also be taken into consideration. For such purpose, reliable prediction of land subsidence is necessary. The Finite Difference (FD) and Finite Element (FE) analyses are generally applied for land subsidence simulation. However, the prediction is rather uncertain due to difficulties in attaining reliable record on historical piezometric drawdown in the underlying soil layer. The other simple prediction of subsidence is based on empirical equations, such as logarithmic equation and linear equation (Duc, 1999).

3. Observational Method for Subsidence Prediction

The analytical solution of consolidation equation for subsidence prediction is not effective under the situation in which soil properties and conditions are quite uncertain. In such case, the observation method is considered to have a great advantage. For one-dimensional consolidation, the AR equation is demonstrated to be a suitable discrete time model for subsidence prediction (Asaoka, 1978).

$$s_j = k_0 + \sum_{i=1}^{n} k_i s_{j-i} \tag{1}$$

where s_j denotes the settlement at the time $t=\Delta t \cdot j$ and the coefficients k_0 and k_i (i=1,2,…,n) are unknown parameters for AR model of order n.

The first and second order models were suggested for subsidence prediction.

4. Estimation by Kalman Filtering

The following linear system equations denote a field equation:

$$x_{t+1} = F_t x_t + G_t w_t \tag{2}$$
$$y_t = H_t x_t + v_t \tag{3}$$

where x_t is a state vector, w_t is a system noise vector, F_t is a state transition matrix, G_t is a driving matrix, y_t is an observation vector, v_t is an observation noise, H_t is an observation matrix, and w_t and v_t are assumed as white Gaussian noises with covariance Q_t and R_t, respectively.

The best and unbiased estimate at time t and error covariance after noisy observation can be expressed as follows:

$$\hat{x}_{t/t} = \hat{x}_{t/t-1} + K_t\left[y_t - H_t \hat{x}_{t/t-1}\right] \tag{4}$$
$$P_{t/t} = P_{t/t-1} - K_t H_t P_{t/t-1} \tag{5}$$

where $\hat{x}$ is an estimate, P is an error covariance, and K is Kalman gain, which can be calculated by the following equation:

$$K_t = P_{t/t-1} H_t^T \left[H_t P_{t/t-1} H_t^T + R_t\right]^{-1} \tag{6}$$

Then, the estimate and error covariance at time t+1 can be obtained as follows:

$$\hat{x}_{t+1/t} = F_t \hat{x}_{t/t} \tag{7}$$
$$P_{t+1/t} = F_t P_{t/t} F_t^T + G_t Q_t G_t^T \tag{8}$$

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Geofront Eng, Dept. of Urban and Environment Eng., Kyoto University, Sakyo-ku, Kyoto 606-8501 Tel. 075-753-5129

5. Identification of Prediction Model Parameters

By considering the subsidence process as stationary, first order and second order AR models were applied to prediction of subsidence (n = 1, 2). The state and observation equations can be expressed as follows:

$$\begin{Bmatrix} k_0 \\ k_1 \\ \vdots \\ k_n \end{Bmatrix}_{t+1} = I \cdot \begin{Bmatrix} k_0 \\ k_1 \\ \vdots \\ k_n \end{Bmatrix}_{t} + I \cdot w_t \qquad (9)$$

$$s_t = \begin{bmatrix} 1 & s_{t-1} & \cdots & s_{t-n} \end{bmatrix} \cdot \begin{Bmatrix} k_0 \\ k_1 \\ \vdots \\ k_n \end{Bmatrix}_{t} + v_t \qquad (10)$$

Subsidence observation data since 1978 to 1997 at ST.10 (Ramkamhaeng University, Central Bangkok) were used in the calculation. Initial value of both noises covariance (Q_t and R_t) and error covariance ($P_{0/0}$) were assumed to be unit matrices.

6. Results and Consideration

From the available data, calculation was set to 18 time steps. The model parameters obtained at each time step after filtering are shown in Fig. 1. The model parameter k_0 converged at early step while k_1 in first order model and k_1 and k_2 in second order model converged at late step (after step 15). The model parameters obtained after convergence were used for prediction. The prediction results are shown in Fig. 2.

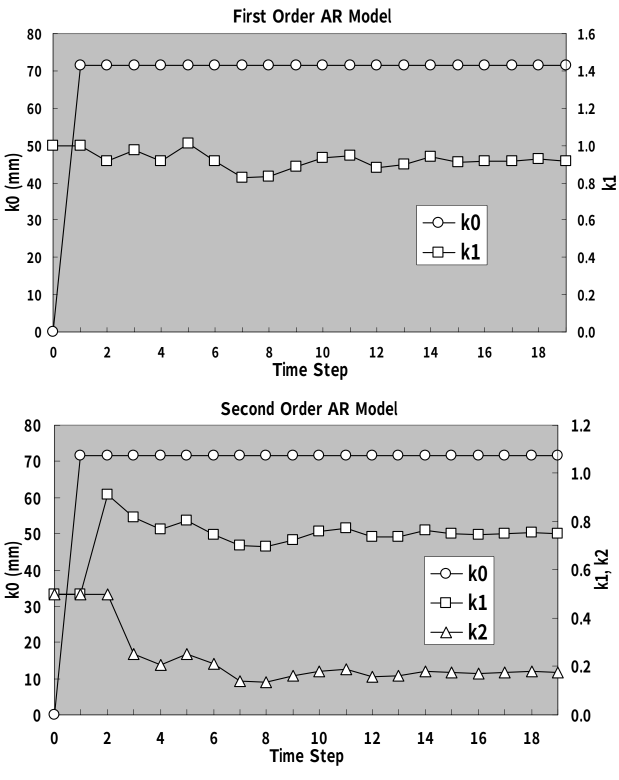

Fig. 1 Model parameters obtained by Kalman filtering

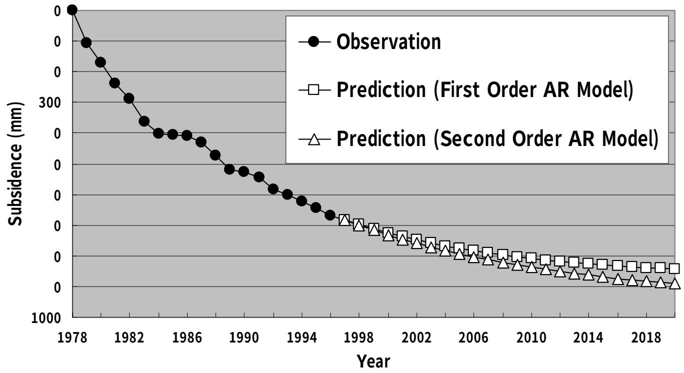

Fig. 2 Subsidence prediction at ST.10

Prospective subsidence from 1997 would reach 160 and 208 mm by 2020 according to the prediction by first order and second order model, respectively.

According to the prediction of 3-D model simulated by JICA et al (1995), in which several future scenarios of groundwater use were estimated for input, the subsidence during 1993-2017 at this area was 200-300 mm in most scenarios. In this study, the prediction for the same period was 233 and 276 mm by using first order and second order model, respectively. It is obvious that the results agree well with that of 3-D model. However, in order to assess the reliability of prediction, further investigation of noise characteristics and estimation error is necessary. The application of observational method using Kalman filtering to other models for subsidence simulation such as FD and FE model is also recommended.

7. Conclusions

This study presents the prediction of land subsidence by observational method using AR model and Kalman filtering. Observed data in Bangkok were used in calculation. Despite rather simple model, prediction result showed a good agreement with the result from 3-D model.

References

Asaoka, A. (1978), *Observational Procedure of Settlement Prediction*, Soils and Foundations, Vol.18, No.4, pp.87-101.

Duc, N. A. (1998), *Updating and Analysis of Bangkok Land Subsidence caused by Deep Well Pumping with Emphasis on Shallow Soil Settlement*, Master Thesis, Asian Institute of Technology, 237 pp.

JICA, DMR and PWD (1995), *The Study on Management of Groundwater and Land Subsidence in the Bangkok Metropolitan Area and Its Vicinity*, Final and Main Report, Kokusai Kogyo Co., Ltd, Japan.

Natalaya, P. and Phienwej, N. (2001), *Subsidence and Flooding in Bangkok*, Asian Institute of Technology, 30 pp.

Ramnarong, V. (1999), *Evaluation of Groundwater Management in Bangkok: Positive and Negative*, Groundwater in the Urban Environment: Selected City Profiles, Rotterdam, pp.51-62.