Updated Lagrangian Analysis of Liquefaction Using Adaptive Mesh Refinement

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1. Introduction

Adaptive mesh refinement(AMR) which has been developed quickly in recent twenty years is one of methods to improve the accuracy of FEM. It can generate fine mesh automaticly according to error level of FEM. The advantages of this method and its adaptability in wide fields have been introduced in many papers. The two procedures, error estimate and mesh refinement, are the key in AMR. Error estimate is the procedure to evaluate the error between the approximate value obtained by FEM and exact value of current elements. In fact, we use a more accurate value than FEM result instead of exact value. Usually, some recovery processes are used here. Mesh refinement is a procedure to refine the mesh indicated by error estimation.

In this research, we apply AMR to a dynamic FEM of liquefaction analysis considering large deformation. A u-p formulation in updated Lagrangian description developed by Di and Sato[1)] is used in FEM equations. An effective elasto-plastic model developed by Oka et al.[2)] based on Biot's tow-phase mixture theory and kinematic hardening rule is usted to simulate liquefaction of soil. A posteriori error estimate procedure developed by Zienkiewicz and Zhu is used and L_2 norm of error of effective stress increment is applied to evaluate the error. Afiision procedure belonging to h-refinement is adopted, when the error of an element exceeds a certain limit, this element is fissioned into four elements. In the numerical example, an embankment standing on liquefiable sand with a strong earthquake input is analyzed, AMR is done during liquefaction process.

2. A posteriori error estimate

A simple posteriori error estimate procedure developed by Zienkiewicz and Zhu[3)] is adopted here. According to least aquare method smoothed stress is calculated by the equation (1). This derived from an energy mean function without weighted in which smoothed element strees σ^* is interpolated by using the same interpolation function N^* to calculate the displacement from nodal stress $\bar{\sigma}^*$ as defined by Eq.(2). σ^h is calculauted by FEM.

$$\int_{\Omega_i} N^{*T}(\sigma^* - \sigma^h)d\Omega = 0 \qquad (1) \qquad \text{and} \qquad \sigma^* = N^*\bar{\sigma}^* \qquad (2)$$

Substituting Eq.(2) into Eq.(1) yields

$$\bar{\sigma}^* = A^{-1}\int_{\Omega_i} N^{*T}\sigma^h d\Omega \qquad (3) \qquad \text{where} \qquad A = \int_{\Omega_i} N^{*T}N^* d\Omega \qquad (4)$$

When calculating Eq.(3), matrix A can be used as a consistent matrix,or a lumped matrix. Then we can get σ^* by Eq.(2). This is in fact a better approximation than σ^h.We use L2 norm to measure a direct error which can be associated with the errors in any quantity. For the strain in the element *i* the L_2 norm of the error e_σ is defined as Eq.(5). Effective stress is selected in liquefaction analysis.

$$\|e_i\| = \left[\int_{\Omega_i}(\sigma^* - \sigma^h)^T(\sigma^* - \sigma^h)d\Omega/\Omega_i\right]^{1/2} = \left[\int_{\Omega_i} e_\sigma^T e_\sigma d\Omega/\Omega_i\right]^{1/2} \qquad (5)$$

In our method, a relative percentage error is used. It is defined for *i*th element as

$$\eta_i = \frac{\|e_i\|}{\|e_i\| + \|\sigma^h\|} \times 100\% \qquad (6)$$

It's a simple method to evaluate errors and easily implemented into the programming code.

3.The *h*-refinement

In the process of AMR, a limit of relative error $\bar{h}$ should be given, after the error estimation, the elements of which error is larger than the limit are refined. *h*-Refinement accuracy of FEM is increased by reducing the size of the elements. A fission scheme[4)] belong to *h*-refinement is used here[4)], in which the selected element is fissioned into four elements. The process is shown in Fig.2. In mesh-a, initial mesh with 6 elements turns to 9 element by fissioning old element 2 into new element 5,7,8 and 9. Five nodes which are nodes from 13 to 17 are generated because there not fissioned

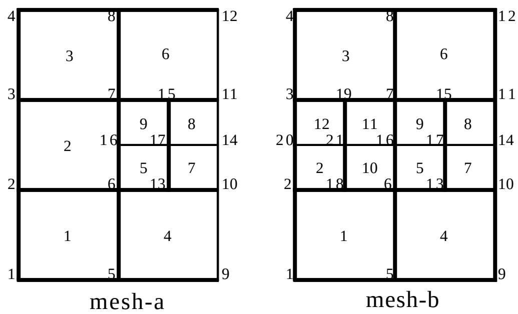

Fig.2. Fission process

キーワード Liquefaction, Adaptive Mesh Refinement, Large Deformation
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new element 2,10,11 and 12, four new nodes which are node 18 to 21 are generated because there is fissioned elements beside right hand side of the old element 2 along the line 6-7 and node 16 was already generated in this side.

When an element is fissioned next to an unfissioned element, slave nodes are created, for example, node 16 of the mesh-a in Figure 2. The motion of a slave node 16 should be governed by the constraint of compatibility, i.e.

$$\mathbf{V}_{16} = \mathbf{T}\{\mathbf{V}_6 \quad \mathbf{V}_7\} \tag{18}$$

where $\mathbf{T}$ is a linear operator which enforces compatibility and $\mathbf{V}_6$ and $\mathbf{V}_7$ are the velocities of the master nodes. When the node 16 is midway between the nodes 6 and 7, $\mathbf{T}$ is defined by [$\mathbf{I}$/2,$\mathbf{I}$/2]. The half part of force in slave node 16 should be added to the master nodes 6 and 7 separately.

4. Numerical examples

We analyze dinamic behavior of an embankment sites on a saturated Edosaki sand layer with Dr=63%, depth of 12m, model length of 56m, shown in Figure 3.(a). Height of embankment is 4.5m, displacement of bottom boundary are fixed, side boundary is fixed in horizontal direction. Acceleration time of recorded at Port Island during the 1995 Hyogoken-Nanbu earthquake shown in Figure.1.(b) is input in horizontal direction. The curve of extra pore water pressure ratio is given in (c). The final mesh is given in Fig.4.

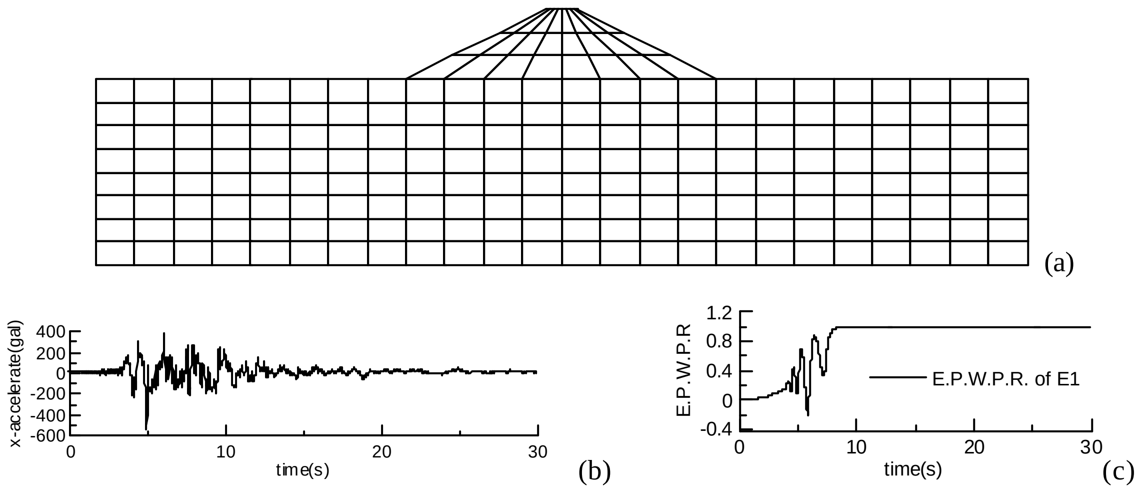

Fig.3. Embankment siting on saturated sand

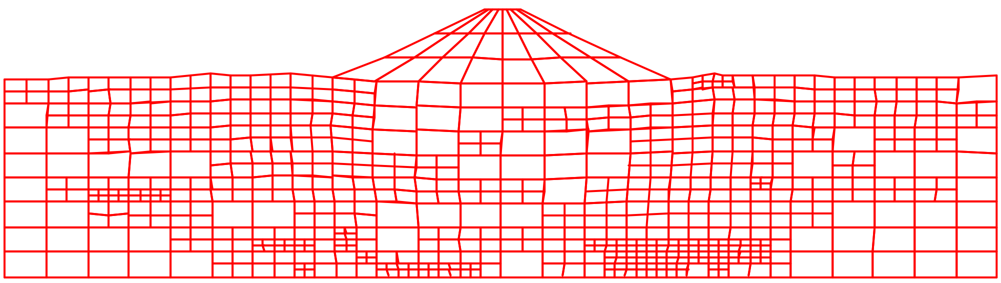

Fig.4. refined mesh at time t=6 sec

5. Conclusion

In this research, AMR is applied in dynamic nonlinear FEM analysis of liquefaction considering large deformation. *h*-Refinement is done indicated by an effective posteriori error estimate. The mesh in the large error area is refined. The advantage is shown in the numerical example.

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