AN ENERGY-BASED MODEL FOR SIMULATING THE UNDRAINED CYCLIC RESPONSE OF SANDS

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1. Introduction

A constitutive model for earthquake site response analysis is proposed, which is based on energy dissipation and simulates the effect of stress dilatancy and cyclic mobility. The model parameters are readily available in geotechnical engineering practice, such as G-γ curves. The new model applies to loose and dense soils and simulates not only the onset of liquefaction, but also the cyclic shear strains that develop after initial liquefaction.

2. Relation between energy dissipated and porepressure

In the case of simple shear test at constant height, dissipated energy is computed as follows using trapezoidal formula,

$$W = \sum_{i=1}^{N-1} (\tau_{i+1} + \tau_i)(\gamma_{i+1} - \gamma_i) \tag{1}$$

where τ_i is the shear stress and γ is the shear strain. As illustrated in Figs. 1a-b, during cyclic undrained simple shear on Nevada sand (D_r = 40%), the mean pressure decreases while the shear strain amplitude increases. However, the shear strain amplitude remains finite due to stress-dilatancy and increase in mean effective pressure.

3. Porepressure buildup and dilatancy model

As proposed by other investigators (e.g., Davis and Berill, 1996), the measured variation of porepressure in Fig. 2 can be fitted using a power relation. Departing from other investigations, we introduce the following relation:

$$u = p_0 (1 - \alpha) \min\left\{1, \left(\frac{W}{W_L p_0}\right)^{\delta}\right\} \tag{2}$$

where α is the parameter controlling the minimum porepressure; W_L is the dimensionless energy at liquefaction; δ is material constants registering the rate of porepressure buildup. Equation 2 has three material constants, namely α, W_L and δ, the value of which are listed in Table 1 for Nevada Sand. The effect of dilatancy is modeled using the concept of phase transformation. When the stress path is above the phase transformation line, i.e., $|\tau| > \sin\varphi_f$, the mean effective pressure becomes:

$$p = p(W) + \left(|\tau| - p(W)\sin\varphi_p\right) / \sin\varphi_f \tag{3}$$

where $p(W) = p_0 - u(W)$ and $u(W)$ is given in Eq. 2, φ_f and φ_p are the angle of friction and phase transformation, respectively. In other terms, changes in porepressure depend only on dissipated energy W for contractant states, and become dependent on shear stress ratios for dilatant states.

4. Stress-strain response during monotonic undrained shear and a single cycle of constant shear strain amplitude

Figure 3 describes a basic construction for simulating an undrained response based on the drained backbones curves (Fig. 3a) corresponding to different mean pressures p_0 but the same G - γ curve. Point A represents the initial state. From point A to B, the liquefaction front moves toward the

Table 1 Material properties for Dr=40 % Nevada sand.

Property	Notation	Value	Unit
Specific gravity	G_s	2.67	
Void ratio	e	0.737	
Failure friction angle	ϕ_f	44	deg
Phase transformation angle	ϕ_p	31	deg
Minimum mean effective pressure ratio	α	0.05	
Dimensionless energy at liquefaction	W_L	0.0025	
Porepressure model exponent	δ	0.5	
Shear modulus at reference pressure p_r	G_r	64	MPa
Reference pressure	p_r	80	kPa
Exponent for shear modulus	n	0.5	

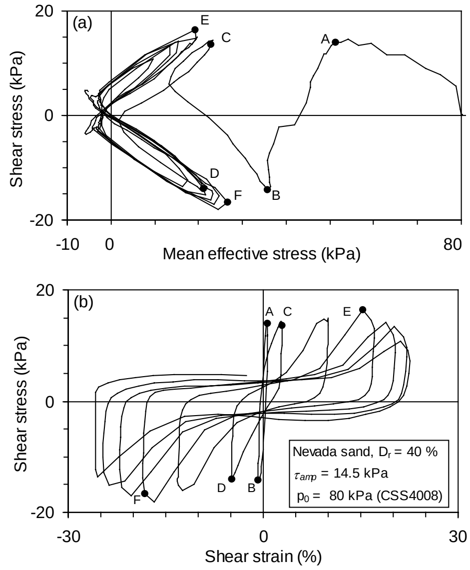

Figure 1 (a) Stress path and (b) stress – strain loop for Nevada sand.

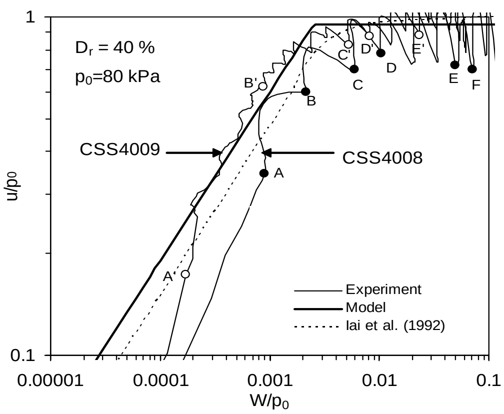

Figure 2 Relation between pore pressure u/p$_0$ and dissipated energy W/p$_0$.

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left. The increase in energy W, which is calculated from the stress-strain response curve (Fig. 3c), creates a change in mean effective pressure (Fig. 3d) and displaces the liquefaction front toward point B (Fig. 3b). At point B, the effective stress path crosses the phase transformation line, and the liquefaction front passes through at point B. From B to C, the effective stress path is parallel to the shear strength line due to stress-dilatancy, the liquefaction front stays at point B. The energy dissipated during loading cycles can be computed from backbone curves after invoking Masing's similitude law. This calculation can be extended for simulating (1) the porepressure buildup during multiple cycles and (2) liquefaction strength curves. The extension is useful to reconcile the liquefaction strength curves based on the thresholds of minimum mean pressure and maximum shear strain amplitude.

5. Stress-strain response during arbitrary loading cycles

Now, we extend the concepts, which were previously developed for monotonic and cyclic loadings of constant amplitude, to arbitrary loading cycles using the constitutive framework proposed by Prevost (1985). Following the approach for pressure-dependent materials, the yield surfaces are assumed to be cones nested in the τ-p space:

$$f_i = |\tau - p\alpha_i| - pR_i = 0 \quad (i=1,\ldots m) \tag{4}$$

where α_i corresponds to the yield surface center and R_i is the slope of the yield surface in the p-τ stress space. The largest yield surface is fixed and corresponds to the failure surface (i.e., $\alpha_m = 0$ and $R_m = \sin\varphi_f$). The parameters α_i, R_i and tangential shear moduli H_i can be constructed from drained backbone curves. The complete algorithm can be found in Tobita (2002). Figure 4 compares simulated results uisng parameter found in Table 1 with those of laboratory test shown in Fig. 1.

6. Conclution

A simplified constitutive model has been proposed to simulate the variations of porepressure and shear strain amplitude of saturated soils subjected to arbitrary cyclic loadings. The model is based on the correlation between dissipated energy W and porepressure buildup u, stress-dilatancy, G -γ curves and hysteretic Masing similitude. The one-dimensional model has been validated by comparing simulated and measured stress-strain responses of Nevada sand subjected to simple shear tests. The proposed model is intended to simulate the response of saturated soils in earthquake site response analysis.

References

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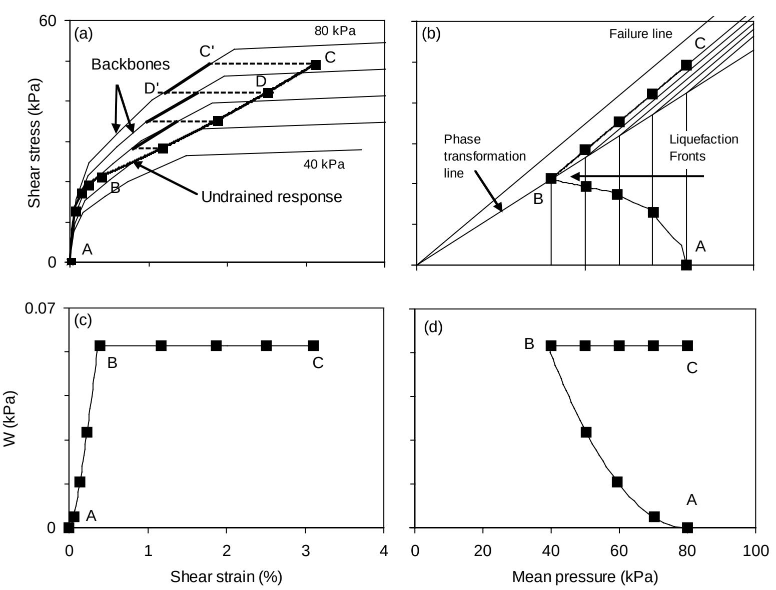

Figure 3 Interpretation of undrained simple shear test based on backbone curves and porepressure model: (a) undrained stress-strain response and drained backbone curves at 40-80kPa; (b) effective stress path and liquefaction fronts; (c) variation of energy with shear strain; and (d) variation of mean pressure with energy.

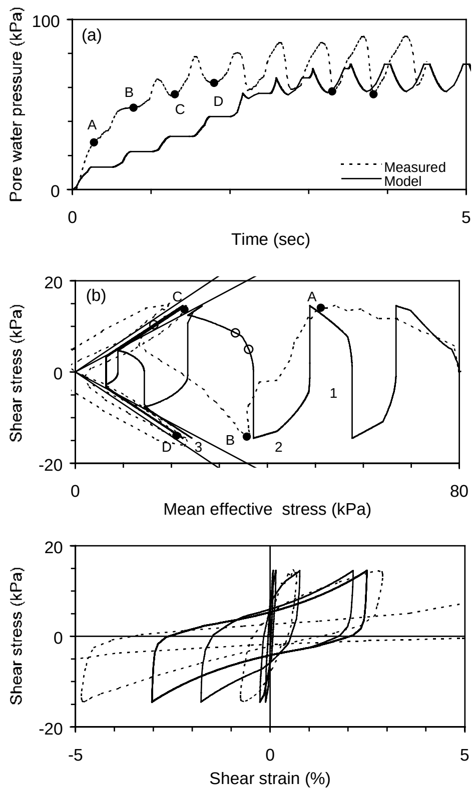

Figure 4 Cyclic simple shear test on Nevada sand: (a) simulated and measured time history of porepressure; (b) effective stress path; (c) stress-strain response (α=0.1).