Rainfall Scaling at High Temporal Resolutions: An Experimental Study

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1. INTRODUCTION

Developments in new stochastic methods based on fractal scaling theory has indicated that the rainfall process can be looked at as one that is independent of any special scales – i.e. exhibiting *scale invariance*. One important consequence of this understanding is the possibility of occurrence of larger extreme events than traditionally accepted, at the finer temporal scales. Among the past scaling studies, the scales below one hour were not as thoroughly analyzed as those above one hour, mainly due to lack of reliable data. The precision of most of the widely used tipping bucket rain gauges make it impossible to construct higher resolution rainfall data free of spurious 'spiking' arising from finite sample size (figure 1).

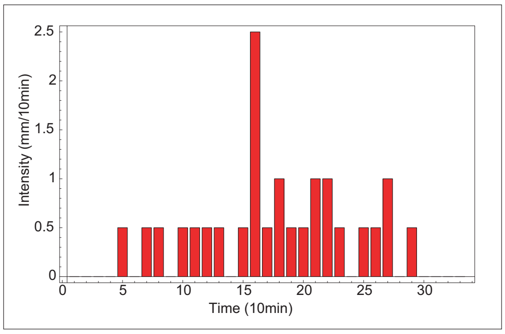

Fig. 1 Spurious spikes at high temporal resolution due to finite size of tipping bucket (.5mm). Source: AMeDAS 10min data.

A rainfall measuring experiment was conducted at several places on the small urban watershed of the *Ebi* river near *Funabashi* in Chiba Prefecture, using 0.1mm tipping bucket rain gauges in order to obtain rainfall data that perform better at small time scales than typical operational rainfall products. This paper presents the results of an investigation of scaling properties of rainfall at scales smaller than 1h using those data.

2. RAINFALL DATA

Tipping bucket data for a continuous period of a year (July 2000 to June 2001) was available for three measuring locations. Since rain gauges with high resolution are susceptible to considerable amount of 'gauge-slip', an intensity correction scheme based on laboratory calibration was employed to process the data before analysis. However, even after the correction, the total annual rainfall amount recorded by those gauges were significantly (about 20%) less than the amounts recorded by 1mm gauges used as controls, indicating that the correction scheme is too conservative. Rainfall time series of 1min resolution were computed from the tipping bucket data and used in the analyses that are presented in the next section.

3. ANALYSIS

Power spectral analysis is a good indicator of fractal scaling. The power spectrum $S(f)$ shows a power-law behavior with wave number, f: $S(f) \sim f^{-\beta}$ in the range of scales where the scaling is present (*scaling regime*). Figure 2 shows the power spectra for the three data sets. They indicate very similar scaling behavior with spectral slopes, β of about 1.1. The scaling regime extends from about 5min up to the largest scale that is possible to be analyzed reliably (about 6-7hours) with the available length of data. Below 5min the spectra flattens-down indicating the end of the scaling regime.

One of the widely used methods for characterizing the nature of multiple scaling is the *codimension* approach (Schertzer and Lovejoy 1987). For a scaling field there is a unique codimension function, $c(\gamma)$, defined by

$$P(R_\lambda \geq \lambda^\gamma) \sim \lambda^{-c(\gamma)} \tag{1}$$

where λ is a non- dimensionalized scale and γ is known as the *order of singularity*. As shown by figure 3, the condimension functions computed for three stations using the range of scales 5min-6h are quite similar.

In order to test the ability of predicting high resolution data from observations made at hourly scale, the following exercise was done: The rainfall data was time-integrated to make 12 hourly rainfall series. Then the resulting series was used to compute the codimension function estimates at the scales of 12, 24, 36, 48 and 64h using the modified form of equation 1: $\log(P(R_\lambda \geq \lambda^\gamma)/b) = -c(\gamma)\log(\lambda)$, where b is a constant to be determined empirically. (It should be noted that the three datasets were treated as an ensemble of the same rainfall process for this calculation, to overcome the estimation problems arising from inadequate length of time-series. This usage is justified by the similarity of scaling properties of three series.) The scale independence of the model was evident from the agreement of the codimension functions at each scale (figure 4). Once the codimension function is established it is possible predict the probability distribution of rainfall intensities at any resolution within the scaling regime. (For the details of the method of codimension function fitting technique that is used here and the simulation of multifractal fields, the reader is referred to Pathirana (2001).) It was attempted to generate the rainfall intensity distribution at 5min scale using the above codimension estimates. The results are compared with the intensities of original rainfall series by means of a quantile-quantile plot in figure 5.

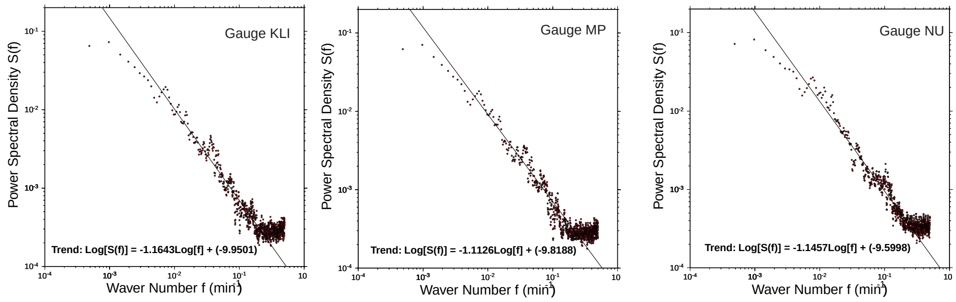

Fig. 2 Power spectra

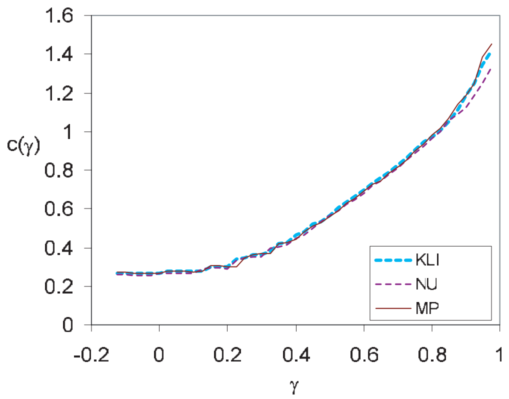

Fig. 3 The codimension functions for three datasets.

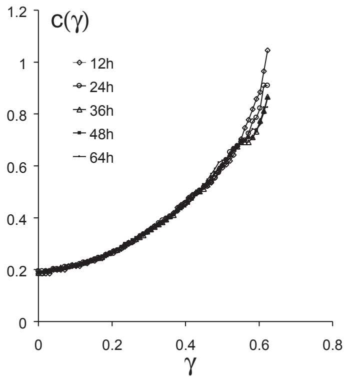

Fig. 4 Codimension functions estimated at larger scales.

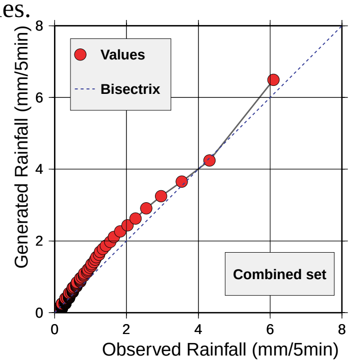

Fig. 5 Comparison of generated and observed intensities.

4. DISCUSSION

A large number of studies involving rainfall scaling in the last two decades has shown that the rainfall process is scaling in space and time dimensions. The temporal scaling has been shown to exist in the regime of scales between 1h to several days. From the results of the present study it can be concluded that the same scaling phenomenon can be extended down to a scale of about 5min. Below 5min resolution, the power spectrum flattens down, indicating a break in scaling. However, it should be noted that with the present data it can not be conclusively stated that the scaling of rainfall process ends at a scale around 5min, because the rainfall time series shows considerable ‘spiking’ at scales like 1-2min, even with the present 0.1mm measuring precision.

The extension of scaling properties to scales below 5min has important implications on studies that are critically dependent on the accurate estimation of rainfall intensities. Examples are estimation of soil erosion and urban storm drainage problems. The existence of scaling implies a multiplicative relationship between different scales and hence, rainfall intensities getting more and more variable as the time scales are reduced. The magnitudes of extreme intensities at these small scales can be much larger than traditionally accepted. Thus, as also suggested by Tsuchiya et al. (2003), the usage of only hourly or larger scale rainfall observations for applications like critical engineering designs is not always appropriate. The present research suggests that it is possible to use fractal theory to estimate rainfall intensity distributions at high resolution using rainfall observations made at hourly or larger intervals.

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